Optimal Corporate Activities under Uncertainty and Competition

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The stage of the research

This dissertation deals with real options approach and game theory. Therefore, two stages are dealt with. Real option is what is applied financial option theory to the world of real assets. Real options approach can analyze optimal investment problem under uncertainty.

An outline of objectives

Commercial enterprises are considered moving for maximizing their expected payoff. Traditional Net Present Value (NPV) method is one of the measures of investment decision. But static NPV method has its limits, since there is a lot of uncertainty in business situations so that actual profit includes a lot of risk. Intuitively, one can see that more alternatives that can react to uncertainty give larger value. Real options show the intuition in mathematics objectively.

Then, their expected payoff is affected variously by the environment where they are active. They must decide timing of decision that maximizes their expected payoff in considering these effects. This dissertation focuses on “uncertainty of profit” and “competition” in these effects. Corporate activities under uncertainty are analyzed by real options approach, and under competition by game theory.

Real options approach is based on financial option theory, so it assumes that there are no interactions between all the agents. In financial market, there are many agents so that individual interactions disappear. However, in actual asset market, there is a lot of strategic interactions. Therefore, it is necessary to consider strategic interactions in real options approach.

On the other hand, corporate activities include some stages depending on the life cycle of the project. First, a firm will enter a new market by initial investment on a new project. Second, it will extend the project when demand increases, and reduce when demand decreases. Or, it will stop the project when demand decreases, and restart when demand recovers. Finally, it exits the market when it can not remain in the market.

Each stage has different real options and games. Therefore, each stage needs to be analyzed individually. This dissertation is intended to analyze optimal corporate activities in each stage under uncertainty and competition by using game theoretic real options approach. This dissertation deals with duopoly to simplify its competition.

Research problem

This dissertation includes following research problems.
1. Entry in duopoly.
2. Extend-reduce in duopoly.
4. Exit in duopoly

“Entry,” “extend-reduce,” “stop-restart” and “exit” represent the stage of life cycles, so they represent the type of real options. “In duopoly” represents the type of competition.

Entry model is analyzed sufficiently in existing literature. Extend-reduce model and stop-restart model are equivalent mathematically. Therefore, this dissertation must deal with stop-restart model and exit model actually.

First, basic model is with symmetric firms and complete information. As extension, there are asymmetric model and incomplete information model. Rewriting, this dissertation includes following research problems.

1. Stop-restart with asymmetric firms in duopoly.
2. Stop-restart with incomplete information in duopoly.
3. Exit with asymmetric firms in duopoly.
4. Exit with incomplete information in duopoly.

**Literature**

Table 1 exhibits literatures that this dissertation should refer to.

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Main Existing literatures in game theoretic real options approach are Grenadier (1996) and Huisman (2001), which are entry model. Especially, Huisman (2001) connects Dixit and Pindyck (1994) and Fudenberg and Tirole (1985).

Grenadier (1996) develops an equilibrium framework for strategic option exercise games. He focuses on a particular example: the timing of real estate development. An analysis of the equilibrium exercise policies of developers provides insights into the forces that shape market
behavior. His model isolates the factors that make some markets prone to bursts of concentrated development. His model also provides an explanation for why some markets may experience building booms in the face of declining demand and property values. While such behavior is often regarded as irrational overbuilding, the model provides a rational foundation for such exercise patterns.

While Grenadier (1996) shows particular phenomenon in real estate market, Huisman (2001) develops pure theoretic models. He considers an investment-timing problem in a duopoly framework. His results of the seminal contribution by Fudenberg and Tirole (1985) are extended by introduction of uncertainty. From the theory of real options it is known that it is optimal to invest when the NPV exceeds the option value of waiting. He modifies the real options investment rule by taking into account strategic interactions. It can be shown that, compared to the option value of waiting in the monopoly case, the strategic option value of waiting is the same in the joint investment case and lower in the preemption equilibrium.

Dixit (1989) analyzes stop-restart decision as “entry-exit.” A firm's entry and exit decisions when the output price follows a random walk are examined. An idle firm and an active firm are viewed as assets that are call options on each other. The solution is a pair of trigger prices for entry and exit. The entry trigger exceeds the variable cost plus the interest on the entry cost, and the exit trigger is less than the variable cost minus the interest on the exit cost. These gaps produce "hysteresis." Numerical solutions are obtained for several parameter values; hysteresis is found to be significant even with small sunk costs.

Myers and Majd (1990) present a general procedure for estimating the abandonment value of a capital investment project. It is easy to think of the abandonment option as an American put option, and somewhat more difficult to put that insight to practical use. They discuss problems of application in some detail and presents numerical estimates of abandonment value for halfway realistic examples.

Fudenberg and Tirole (1986) develop a duopoly model in which exit occur because of the existence of fixed costs or opportunity costs. Each firm enters the market knowing its own cost, but not that of its opponent. As times goes on, each firm becomes increasingly pessimistic about the cost of its remaining rival. The time of exit is the only strategic variable, so that our model is a "war of attrition." In contrast to the classic war of attrition, however, they assume that with positive probability each firm's costs may be low enough that staying in forever is a dominant strategy. Thus their model, unlike the classic one, has a unique equilibrium.

**Methodology**

In game theoretic real options approach, first we derive value functions by using real options approach, second equilibria by game theory. Especially, this dissertation considers a “timing game.”
In a simple timing game, each player’s only choice is when to choose the action “stop,” and once a player stops he has no effect on future play. Therefore, we must choose appropriate real options and timing games.

When we derive value function, we should maximize the conditional expectation of value of the firm with respect to stopping time. At the “stopping time,” a firm decides to enter or exit, so that the maximizing problem should be solved as the optimal stopping problem. To solve the optimal stopping problem, we need ideas of stochastic calculus and optimal control.

For stop-restart model, the appropriate real option is the option introduced by Dixit (1989), but the timing game is not found. It is necessary to find an appropriate timing game for stop-restart.

For exit model, the appropriate real option is American put option, and the timing game is the war of attrition. This dissertation has already construct exit model with symmetric firms and complete information. It is easy to introduce asymmetry, but not incomplete information.

Fudenberg and Tirole (1986) present the exit model with incomplete information and no uncertainty. So, introducing uncertainty to Fudenberg and Tirole (1986) presents the exit model with incomplete information. It is easy to derive value functions by using real options approach, however, it is hard to derive equilibria because the partial differential equation to find contains insoluble terms. It is necessary to find reasonable assumptions or another solution.

**Expected outcome**

It is clear that uncertainty put off the decision by NPV method. Furthermore, Huisman (2001) shows competition hastens the decision by real options approach, but the decision is later than that by NPV method.

In exit model with symmetric firms and complete information, the result is equivalent to Huisman (2001). Incomplete information may hasten the decision by complete information. Possibly, the decision by highly incomplete information may be faster than that by NPV method. In stop-restart model, the result will be also equivalent to exit model.

Summarized, at every stage of life cycles, the decision by NPV method is the fastest, the second fastest is that by game theoretic real options approach, and the latest is that by real options approach. However, highly incomplete information possibly hastens the decision by NPV method.

**References**


