Channel Performance Under Consignment Contract with Revenue Sharing

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Under a consignment contract with revenue sharing, a supplier decides on the retail price and delivery quantity for his product, and retains ownership of the goods; for each item sold, the retailer deducts a percentage from the selling price and remits the balance to the supplier. In this paper we show that, under such a contract, both the overall channel performance and the performance of individual firms depend critically on demand price elasticity and on the retailer’s share of channel cost. In particular, the (expected) channel profit loss, compared with that of a centralized system, increases with demand price elasticity and decreases with retailer’s cost share, while the profit share extracted by the retailer decreases with price elasticity and increases with retailer’s cost share. With an iso-price-elastic demand model, we show that the channel profit loss cannot exceed 26.4%, and that the retailer’s profit share cannot be below 50%. When price elasticity is low, or when the retailer’s cost share approaches 100%, or both, the retailer can extract nearly all the channel profit that is almost equal to the centralized channel profit.

Key words: consignment sales; revenue sharing; supply chain management

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1. Introduction

Amazon.com has an online “marketplace” where anyone can list for sale new, used, or refurbished items (books, CDs, electronics, tools and hardware, kitchen and housewares, etc.). With a few minor restrictions, sellers decide on how many units to list and the items’ selling price. The listing itself is free. Amazon.com charges sellers according to the following policy, which is essentially a consignment contract with revenue sharing:

Amazon.com collects a fee only when your item sells. At that time, Amazon.com collects your sales price from the buyer, deducts a commission of $0.99 plus 15% of the sales price (10% for Electronics and Camera & Photo items), and deposits the rest in your account. (Note: Pro Merchant Subscribers don’t pay the $0.99 commission.) . . . If your item doesn’t sell within 60 days, the listing is closed and you pay nothing . . .

(For more details on Amazon.com’s marketplace, go to: http://www.amazon.com.) Adopting a similar market structure, in 2003 America Online is set to launch an Internet marketplace targeting small- and medium-size businesses selling consumer goods (Wingfield and Angwin 2002).

Consignment with revenue sharing as a business arrangement is well documented in business textbooks:

Under such an arrangement, ownership of the goods is retained by the supplier. Price is also usually determined solely by the resource (the supplier). For each item sold, the retailer will deduct an agreed percentage from the selling price and remit the balance to the vendor. No money changes hands until the item is sold. (Bolen 1978, p. 199)

A consignment arrangement with revenue sharing naturally favors the retailer. Since no payment to the supplier is made until the item is sold, the retailer has no money tied up in inventory and bears no risk associated with demand uncertainty. That the supplier can choose and impose a delivery quantity may sound unfavorable to a retailer in cases where the retailer may have limited or expensive shelf space and inventory handling costs. The retailer, however, usually determines the terms of the contract, i.e., the percentage allocation of sales revenue between herself and the supplier. Thus, as we show later with an iso-price-elastic and multiplicative demand model, she always designs the contract, based on her own
cost structure and that of the supplier, in such a way that the supplier never delivers quantities that are not in the retailer’s interest. The retailer’s optimal contract would be a two-part contract, passing along all variable costs and revenues to the supplier and using a fixed fee to extract all channel profits. However, such contracts can be difficult to implement in practice, for reasons we discuss in more detail at the end of the paper. Here, we limit ourselves to revenue-sharing contracts in §6.

The purpose of this paper is to investigate the decisions and channel performance under consignment contract with revenue sharing. We ask the following key questions:

1. How do the firms interact in the channel and how do they make their individual decisions in equilibrium?
2. How does a decentralized channel perform, compared with a centralized channel, and how do system parameters affect channel performance?
3. How do individual firms perform? Or, how are the total channel sales revenue and profit distributed between the supplier and the retailer? How do system parameters affect individual firms’ performance?

We perform our investigation under the following channel setting. An upstream manufacturer produces a product at a constant marginal cost. He then sells the product through an independent retailer. The retailer does not pay the manufacturer upon receipt of the items but shares the sales revenue on units sold. The retailer incurs a per-unit cost on delivered items (e.g., shelf space and inventory handling costs, etc.). The product has a single selling period or season (e.g., the 60-day time window in the Amazon.com example), and the manufacturer has only one chance at production before the start of the season. Total demand during the season is both price sensitive and uncertain.

We model the decision making of the two firms as a Stackelberg (leader-follower) game: The retailer, acting as the leader, offers the manufacturer a take-it-or-leave-it contract, which specifies the percentage allocation of sales revenue between herself and the manufacturer. The manufacturer, acting as a follower, chooses how many units of the product to produce and the retail price. The manufacturer accepts the contract as long as he can earn a positive profit. (That is, we normalize the manufacturer’s reservation profit to zero.)

The above supply chain model, though simple, is rich enough to capture the key trade-offs and interactions of the two firms in their decision making. Although the retailer has the freedom to allocate to herself a large share of the revenue, the manufacturer will trade-off his share of the revenue against his production costs in choosing the production quantity and retail price. Thus, leaving the manufacturer a small revenue share may induce him to choose a low production quantity and to insist on a high retail price, which may harm both parties.

Our analyses first assume a specific demand function form having a deterministic, iso-price-elastic curve multiplied by a random factor (e.g., Petruzzi and Dada 1999). Under a very mild restriction on the distribution function of the random factor, we provide a complete characterization of the equilibrium solution for this seemingly complex Stackelberg gaming problem. The solution then allows us to quantify explicitly, and gain insights into, the channel performance. Specifically, we derive the optimal revenue-share allocation for the retailer and show that it has a surprisingly simple, closed-form solution. We show that both the overall channel performance and that of individual firms depend critically on two system parameters: the price elasticity of demand, and the retailer’s share of the channel cost. In particular, the channel profit loss, which is defined as the profit difference in percentage between a centralized channel and a decentralized channel, increases with demand price elasticity and decreases with the retailer’s share of cost. With the iso-price-elastic and multiplicative demand model, however, the loss usually cannot exceed around 26.4%; it approaches zero either when the price elasticity is very low (close to one) or when the retailer bears almost all the channel cost. The profit share extracted by the retailer in the channel decreases with demand price elasticity and increases with the retailer’s cost share, and cannot be lower than 50%. When the price elasticity is low, or when the retailer’s share of cost approaches 100%, or both, the retailer can extract nearly all (i.e., close to 100%) of the decentralized channel profit. Under those circumstances the decentralized channel profit is nearly equal to the centralized channel profit. Thus, when product demand is very insensitive to price, a consignment contract can be extremely efficient and desirable from the point of view of a retailer.

We derive the above properties and insights analytically. However, the analyses are done under a specific assumption, i.e., the iso-price-elastic and multiplicative demand function form. To test the generality of those properties and insights, we then consider a different form of demand function—one with a linear demand curve plus a random variable. For this linear and additive model, we prove the existence of a unique equilibrium solution for decentralized decisions. While analytical characterization of channel performance turns out to be impossible due to the complexity of the associated analyses, our numerical studies suggest that those properties and insights derived analytically based on the iso-price-elastic model may hold for the linear and other general demand models.
The rest of the paper proceeds as follows. Section 2 provides a brief review of related literature. Section 3 details the model assumptions and derives the centralized decisions. Section 4 characterizes the decentralized decisions and performance for the iso-price-elastic demand model, and §5 considers the linear demand model. Section 6 concludes the paper with future research directions. All mathematical proofs are found in the appendix.

2. Related Literature
The model setting we consider in this paper has (or is a combination of) three distinctive features: (1) a revenue-sharing scheme offered by the downstream retailer, (2) a production or inventory quantity and retail price decision made by the upstream supplier, and (3) inventory consignment, i.e., full ownership of inventory retained by the supplier. In the following, we provide a brief review of papers that relate to these model features.

Cachon and Lariviere (2001) study supply chains where a downstream manufacturer facing an uncertain demand offers various contracts to an upstream component supplier to motivate the supplier to build up production capacity. One of the contracts considered is where the downstream manufacturer offers a price for purchasing components from the upstream supplier. In their model the retail price of the product is fixed exogenously, hence the purchasing price offered to the supplier represents a “share” of the selling price or the sales revenue. Thus, the above contract can be regarded as a revenue-sharing scheme. In the current paper our model extends Cachon and Lariviere’s model by adding a price dimension to the decision of the upstream firm.

The paper by Gerchak and Wang (2003) is more closely related to our setting of consignment with revenue sharing. Gerchak and Wang consider assembly systems, where the firm assembling the final product chooses the allocation of sales revenue between herself and multiple suppliers, each producing a different component needed for the final product, and the suppliers then decide on their individual component production quantities. They derive the equilibrium revenue-sharing allocation and production quantities. Wang and Gerchak (2003) then extend that model into a setting where the decisions are about production capacities, rather than upfront production quantities. In both papers, however, the retail price of the final product is assumed to be a constant.

Revenue sharing as a means of business arrangement has also been implemented under settings other than consignment. For example, in the video rental industry a supplier charges retailers an upfront wholesale price plus a share of the sales revenue. A downstream retailer then chooses the order quantity or the retail price, or both. Here it is the upstream supplier who determines the terms of the contract. Cachon and Lariviere (2002) demonstrate that using such a contract, a supplier can coordinate a single retailer channel. (Cachon and Lariviere’s argument also applies to our consignment setting. That is, if the retailer pays the supplier an upfront price in addition to a share of sales revenue, she can then coordinate the channel.) Dana and Spier (2001) study this contract in the context of a perfectly competitive market faced by downstream retailers. Other related papers include Mortimer (2000), Pasternack (2000), and Gerchak et al. (2001).

Our specific modeling aspect that the upstream supplier makes channel production or inventory decision is similar to a vendor-managed inventory (VMI) program, e.g., Aviv and Federgruen (1998), Fry et al. (2001), and references therein. In reality, when implementing such a program, a downstream retailer may impose various constraints on the supplier’s inventory decision, such as a minimum customer demand fill-rate service level, or a control range on inventory stocking levels; see Fry et al. (2001) for industry examples and more detailed discussions.

From the viewpoint of inventory ownership, a pure consignment contract such as ours specifies that the supplier retains full ownership of inventory, and, hence, bears all the risk associated with overstocking. A pure wholesale-price contract would be the other extreme, i.e., the downstream retailer owns the inventory and bears the full risk of overstocking. In between is an inventory buyback or return policy where overstocking risks are shared. A number of authors focus on models to illustrate the effect of shared inventory ownerships on channel performance. Rubinstein and Wolinsky (1987) compare consignment versus nonconsignment contracts in a setting with multiple buyers, sellers, and middlemen. Hackett (1993) considers consignment contracts where the retailer exerts a sales effort. Kandel (1996) studies factors that affect the choice of inventory return policy. Pasternack (1985) shows that a properly designed inventory return policy can achieve channel coordination in a newsvendor setting. Emmons and Gilbert (1998) explore the effect of inventory return on channel performance when a downstream retailer makes both price and production decisions.

In our revenue-sharing contracting model, the revenue share set by the retailer interacts with the retail price, and hence the total channel profit margin, chosen by the supplier. Such a model setting differs from most channel models found in the marketing literature. In those models, firms usually interact with each other by setting their individual profit margins; see, for example, Jueland and Shugan (1983), Lal and...
Assume that the probability distribution has support on $[A, B]$ with $B > A > 0$ and so $\mu > 0$. Let $h(x) \equiv f(x)/(1 - F(x))$ denote the failure rate function of the demand distribution. We let $y(p)$ take the form of

$$y(p) = ap^{-b} \quad \text{where } a > 0, b > 1. \quad (2)$$

The above demand function form is one of the few models that have often been adopted in the literature for studying joint pricing-inventory decisions; see Petruzzi and Dada (1999) for an excellent review and extensions. In this formulation, the parameter $b$ is the price-elasticity index of (expected) demand. The larger the value of $b$, the more sensitive the demand is to a change in price. If the price-elasticity index is greater than 1, then a product is defined as price elastic; if the price-elasticity index is 1 or less, then a product is defined as inelastic. We focus on price-elastic products by assuming $b > 1$. (If $b < 1$, we can show that the optimal price for the optimization problem to be considered later goes to infinity.) A second demand function often adopted in the literature has a form that is linear in price and additive in randomness, that is, $D(p) = y(p) + \epsilon = a - bp + \epsilon$. We consider this demand function in §4.

The product is produced at a constant cost of $c_M$ per unit, and there is a cost of $c_R$ per unit incurred at the retail stage for inventory handling, shelf-space usage, etc. Equivalently, we define $c \equiv c_M + c_R$ as the total channel cost per unit, out of which $\alpha \equiv c_R/c$ portion is incurred at the retail stage and the rest, i.e., $1 - \alpha = c_M/c$, at the production stage. For simplicity, we assume that any unsold product at the end of the season bears no salvage value or disposal cost. Similarly, in the case of shortages, unsatisfied demand carries no additional penalty except for the loss of sales revenue. For seasonal or short life-cycle products, zero salvage value or holding cost and zero shortage penalty assumptions are appropriate reflections of reality.

To maximize the expected channel profit, a central decision maker must simultaneously choose the retail price and the production quantity for the product. This decision has to be made before the decision maker can observe the demand realization. Let $\Pi_i(p, q)$ denote the expected channel profit for any chosen price $p$ and production quantity $q$. We have

$$\Pi_i(p, q) = -cq + pE[min(q, D)]$$
$$= -cq + pE[min(q, y(p)e)]. \quad (3)$$

Following Petruzzi and Dada (1999), we define $z \equiv q/y(p)$, and call it the “stocking factor” of inventory. Then, the problem of choosing a price $p$ and a production quantity $q$ is equivalent to choosing a price $p$ and a stocking factor $z$. By substituting $q = zy(p)$ into (3), the objective function can be rewritten as

$$\Pi_i(p, z) = -czy(p) + p[y(p)E[min(z, e)]]$$
$$= y(p)[p(z - \Lambda(z))] - cz], \quad (4)$$

where $y(p) = ap^{-b}$ and

$$\Lambda(z) = \int_A^z (z - x)f(x) \, dx. \quad (5)$$

The above transformation of variables will allow us to write the optimality conditions more conveniently. We follow a sequential procedure to find the optimal solution, denoted by $(p^*_z, z^*_z)$, that maximizes $\Pi_i(p, z)$ of (4). That is, we find the optimal price $p^*_z(z)$ for any fixed $z$, and then maximize $\Pi_i[p^*_z(z), z]$ over $z$ to find $z^*_z$.

**Theorem 1.** For any fixed $z$ such that $A \leq z \leq B$, the unique optimal price $p^*_z(z)$ is given by

$$p^*_z(z) = \frac{bc}{b - 1} \cdot \frac{z}{z - \Lambda(z)}, \quad (6)$$

and, if $d[xh(x)]/dx = h(x) + x dh(x)/dx > 0$, the optimal $z = z^*_z$ that maximizes $\Pi_i[p^*_z(z), z]$ is uniquely determined by

$$F(z^*_z) = \frac{z^*_z + (b - 1)\Lambda(z^*_z)}{bz^*_z}. \quad (7)$$

All proofs are in the appendix.

For a more general problem with linear overage and shortage costs, Petruzzi and Dada (1999) establish a sufficient condition, for the uniqueness of the optimal solution, that requires $2h(x)^2 + dh(x)/dx > 0$ and $b \geq 2$. Note that both $2h(x)^2 + dh(x)/dx > 0$ and $d[xh(x)]/dx = h(x) + x dh(x)/dx > 0$ put very mild restrictions on the demand distribution. Obviously, they are both implied by the IFR (increasing failure rate) condition, a property that is known to be satisfied by distributions such as normal, uniform, as well as the gamma and Weibull families, subject to parameter restrictions (Barlow and Proschan 1965). An advantage of our Theorem 1 here is that it does
4.1. Manufacturer’s Problem

The following proposition describes how the optimal decision \((p^*_r, z^*_r)\) changes with system parameters.

**Proposition 1.** If \(d(xh(x)) = h(x) + xdh(x)/dx > 0\), then:

1. \(z^*_r\) is decreasing in \(b\), but is not affected by \(c\);
2. \(p^*_r\) is increasing in \(c\), and is decreasing in \(b\).

Substituting \((p^*_r, z^*_r)\) of \((6)\) and \((7)\) into \((4)\), we can write the optimal system profit as

\[
\Pi^*_r = \frac{ac}{b-1} (p^*_r)^b z^*_r. \quad (8)
\]

We can show that \(\Pi^*_r\) increases in \(a\) and decreases in \(c\). However, the way by which \(\Pi^*_r\) depends on \(b\) is more complex and is not monotone in general.

As an example we consider the case where the random factor \(e\) follows a uniform distribution on \([0, B]\). Then, we have \(f(x) = 1/B, F(x) = x/B\), and \(\Lambda(z) = z^2/2B\). From \((6)\) and \((7)\), the optimal solution is given by

\[
p^*_r = ((b+1)/(b-1)) \cdot c, \quad z^*_r = 2B/(b+1), \quad \text{and so,}
\]

\[
q^*_r = a(p^*_r)^b z^*_r = \frac{2ab(b-1)^b}{(b+1)^{b+1}c^b}.
\]

Substituting \((p^*_r, z^*_r)\) into \((8)\), we obtain the maximum system profit as

\[
\Pi^*_r = \frac{2ab \cdot (b-1)^{b-1}}{c^{b+1} \cdot (b+1)^{b+1}}.
\]

**4. Decentralized Channel with the Iso-Price-Elastic Demand Model**

In the decentralized channel, a manufacturer produces the product and then sells it to market through a retailer under consignment. The retailer offers the manufacturer a revenue-sharing contract which stipulates that for each unit of the product sold, she keeps \(r\) share (percent) of the revenue for herself and remits the rest, i.e., \(1-r\), to the manufacturer. No money changes hands unless an item is sold. The manufacturer has the right to decide on the retail price \(p\) and on the production quantity \(q\) for his product.

### 4.1. Manufacturer’s Problem

For a given revenue share \(r\), \(0 \leq r \leq 1\), allocated by the retailer, the manufacturer’s problem is to choose the retail price and the production quantity to maximize his own expected profit. As in §3, we define the stocking factor of inventory as \(z \equiv q/y(p)\). Then, choosing \((p, q)\) is equivalent to choosing \((p, z)\). Denote the manufacturer’s profit function by \(\Pi_{d, M}(p, z)\). We have

\[
\Pi_{d, M}(p, z) = -(1-\alpha)cq + (1-r)pE[\min(q, D)]
\]

\[
= y(p)[(1-r)p[z - \Lambda(z)] - (1-\alpha)cz], \quad (9)
\]

where \(y(p)\) and \(\Lambda(z)\) are defined in \((2)\) and \((5)\), respectively.

The optimization problem in \((9)\) has a structure similar to that of the centralized problem in \((4)\), and under the same assumption about the demand distribution as that in Theorem 1, we can show that the unique optimal policy for the manufacturer, denoted by \((p_d, z_d)\), satisfies the first-order conditions. That is, for any fixed \(z_d\), the unique optimal price \(p_d\) for the manufacturer is given by

\[
p_d = \frac{bc}{b-1} \cdot \frac{z_d - \Lambda(z_d)}{1-\alpha}, \quad (10)
\]

and the unique optimal \(z_d\) satisfies

\[
F(z_d) = \frac{z_d + (b-1)\Lambda(z_d)}{bz_d}. \quad (11)
\]

The following proposition characterizes the decentralized decision \((p_d, z_d)\) and its relationship to the centralized decision \((p^*_r, z^*_r)\).

**Proposition 2.** In the decentralized channel,

1. the manufacturer’s optimal stocking factor \(z_d\) does not depend on the revenue share \(r\), the channel cost \(c\), and the retailer’s cost share \(\alpha\), and is always equal to that of the centralized system, i.e., \(z_d = z^*_r\);
2. the manufacturer’s optimal price depends on the retailer’s revenue share \(r\), the channel cost \(c\), and the retailer’s cost share \(\alpha\), and relates to the centralized price \(p^*_r\) through \(p_d = p^*_r(1-\alpha)/(1-r)\).

The above properties seem to depend heavily on the iso-price-elastic demand function form. For example, for the linear demand function to be considered in §5, we could not establish these simple mathematical relationships between the decentralized and centralized decisions.

### 4.2. Retailer’s Decision

Knowing that the manufacturer chooses \((p_d, z_d)\) according to \((10)\)–\((11)\) in response to a given revenue share allocation \(r\), the retailer decides on \(r\) to maximize her own profit. The retailer’s profit function, denoted by \(\Pi_{d, R}(r)\), is given by

\[
\Pi_{d, R}(r) = -\alpha cz_d + rp_dE[\min(q_d, D)]. \quad (12)
\]

Substituting \(q_d = y(p_d)z_d\) and \(D = y(p_d)\varepsilon\) into \((12)\), we have

\[
\Pi_{d, R}(r) = y(p_d)\{rp_d[z_d - \Lambda(z_d)] - \alpha cz_d\}. \quad (13)
\]

Since \(y(p_d) = a(p_d)^{-b}\) and \((p_d, z_d)\) are determined through \((10)\)–\((11)\), we can further write

\[
\Pi_{d, R}(r) = \frac{a(b-1)^{b-1}}{b(1-\alpha)^{b-1}} \cdot \frac{[z_d - \Lambda(z_d)]^b}{(z_d)^{b-1}} \cdot g(r), \quad (14)
\]
where,
\[
g(r) = \left\{ \left[ (1 - \alpha)b + (b - 1)\alpha \right] r - (b - 1)\alpha \right\} \\
\cdot (1 - r)^{b-1}.
\] (15)

Recall that \( z_d \) chosen by the manufacturer does not depend on \( r \). Thus, maximizing \( \Pi_{d,k}(r) \) over \( r \) is equivalent to maximizing the function \( g(r) \).

**THEOREM 2.** The optimal revenue share allocation for the retailer is unique, and is given by
\[
r^* = \frac{a(b - 2) + 1}{b - \alpha}.
\] (16)

The retailer’s optimal revenue share \( r^* \) in (16) depends on nothing but two system parameters, the demand price-elasticity index \( b \), and the retailer’s cost share \( \alpha \). The following proposition characterizes how \( r^* \) changes with \( \alpha \) and \( b \).

**PROPOSITION 3.** For \( 0 < \alpha < 1 \) and \( b > 1 \), the retailer’s optimal revenue share \( r^* \) is

1. strictly larger than her cost share \( \alpha \);
2. increasing in \( \alpha \) for given \( b \), and decreasing in \( b \) for given \( \alpha \).

Part (1) of Proposition 3 states that in equilibrium the retailer always allocates to herself a share of the revenue that is strictly larger than her share of the channel cost. That the retailer’s revenue share is increasing in her cost share is rather intuitive. The following is an intuitive explanation for the fact that the retailer’s revenue share is increasing in \( b \): Note from (10) that the retail price chosen by the manufacturer is increasing in retailer’s revenue share \( r \). The larger the price-elasticity index the more dramatic the decrease in channel demand with an increase in retail price. The more dramatic the decrease in channel demand the more dramatic the decrease in total channel sales revenue. The retailer cares about her net profit. As a consequence, when the price elasticity is high, it is actually beneficial for her to be less greedy in allocating the revenue share.

**REMARKS.** In equilibrium, the retailer allocates \( r^* \) share of the sales revenue for herself and \( 1 - r^* \) to the manufacturer. Accordingly, the manufacturer sets the price \( p^*_d = ((1 - \alpha)/(1 - r^*))p^*_c \), since \( r^* > \alpha \), and delivers the quantity \( q^*_d = y(p^*_d)z^*_c < q^*_c \) for the channel. With \( r = r^* \), while the pair \((p^*_d, q^*_d)\), or equivalently \((p^*_c, z^*_c)\), maximize the manufacturer’s profit function of (9), it does not in general maximize the retailer’s profit function of (13). Then, what would be the price and quantity that maximize the retailer’s profit, and how do they compare with \((p^*_d, q^*_d)\)? Let \((p^*_c, z^*_c)\) denote the pair that maximizes retailer’s profit. From (13), we can show that \( z^*_c = z^*_c \) and \( p^*_c = (\alpha/r^*)p^*_c < p^*_c < p^*_d \). As a consequence, the optimal quantity for the retailer would be \( q^*_c = y(p^*_c)z^*_c > q^*_c > q^*_c \). In other words, the delivery quantity (selling price) chosen by the manufacturer in the channel would be too low (too high) for the retailer. Consequently, under the consignment contract with revenue sharing and with the iso-price-elastic and multiplicative demand model, it is never in the interest of the retailer to reject any portion of the quantity delivered by the manufacturer.

### 4.3. Performance Implications of Decentralized Decisions

Total profit of the channel depends on nothing but the retail price and the production quantity. When a decentralized channel reaches the same decision as a centralized system, i.e., when \((p_d, z_d) = (p^*_c, z^*_c)\), the decentralized channel achieves the same channel profit as the centralized system. In that case we say that the decentralized channel is coordinated. From the relationship between the decentralized decision \((p_d, z_d)\) and the centralized decision \((p^*_c, z^*_c)\) in Proposition 2, we see that the only case where \((p_d, z_d) = (p^*_c, z^*_c)\) is when \( r = \alpha \), i.e., when the retailer allocates to herself a revenue share that is exactly equal to her share of the channel cost. However, Part (1) of Proposition 3 states that in equilibrium the retailer always allocates to herself a revenue share that is strictly larger than her cost share, i.e., \( r > \alpha \). As a consequence, it follows from Part (2) of Proposition 2 that \( p_d > p^*_c \). The production quantity is given by \( q_d = y(p_d)z_d \). Since \( y(p) \) is a decreasing function and \( z_d = z^*_c \), it then follows that \( q_d = y(p_d)z_d < y(p^*_c)z^*_c = q^*_c \).

**COROLLARY 1.** The retail price in the decentralized channel is always higher than that in a centralized system, and the production quantity is always lower. As a consequence, the decentralized channel profit is always lower than the centralized channel profit.

For the rest of this subsection we quantify the decision and profit gaps between a centralized channel and a decentralized channel, and then study how the decentralized channel profit is distributed between the retailer and the manufacturer. We show that all the differences depend on two key system parameters—the demand price-elasticity index \( b \), and the retailer’s share of channel cost \( \alpha \). We derive a lower bound for the decentralized channel performance and a lower bound for the retailer’s share of the channel profit.

First, we look at how the price and production quantities in a decentralized channel deviate from their centralized counterparts. Let \((p^*_c, z^*_c)\) denote the equilibrium retail price and the stocking factor in a decentralized channel. Since the retailer sets \( r^* = (a(b-2)+1)/(b-\alpha) \), it follows from Part (2) of Proposition 2 that
\[
p_d^* = \frac{1 - \alpha}{1 - r^*}, \quad p_c^* = \frac{b - \alpha}{b - 1} p_c^* > p_c^*.
\] (17)
where the inequality follows since \( b > 1 \) and \( 0 < \alpha < 1 \). The decentralized production quantity is given by \( q_d^* = y(p_d^*)z_d^* = a(p_d^*)^{-b}z_d^* \). Substituting (17) for \( p_d^* \) and using the fact that \( z_d^* = z_c^* \), we have

\[
q_d^* = \left( \frac{b-1}{b-\alpha} \right)^b \cdot a(p_d^*)^{-b}z_c^* = \left( \frac{b-1}{b-\alpha} \right) q_c^* < q_c^*. \tag{18}
\]

Let \( \Delta_p \equiv (q_d^* - p_c^*)/p_c^* = (1 - \alpha)/(b-1) > 0 \) and \( \Delta_p \equiv (q_d^* - q_c^*)/q_c^* = [(b-1)/(b-\alpha)]^b - 1 < 0 \) be the (percentage) deviations of price and production quantity, respectively. The following proposition characterizes how they each change with the price-elasticity index \( b \) and the retailer’s cost share \( \alpha \).

**Proposition 4.**

1. \( \Delta_p \) is decreasing both in \( b \) and in \( \alpha \). For any given value of \( \alpha \), \( \Delta_p \) decreases from \( \infty \) to 0 as \( b \) increases from 1 to \( \infty \); and for any given value of \( b \), \( \Delta_p \) decreases from \( 1/(b-1) \) to 0 as \( \alpha \) increases from 0 to 1.

2. \( |\Delta_p| \), the absolute value of \( \Delta_p \), is decreasing both in \( b \) and in \( \alpha \). For any given value of \( \alpha \), \( |\Delta_p| \) decreases from \( 1 \) to \( 1/(1-e^{-\alpha}) \) as \( \alpha \) increases from 0 to \( \infty \); and for any given value of \( b \), \( |\Delta_p| \) decreases from \( 1 - [1 - 1/b]^b \) to 0 as \( \alpha \) increases from 0 to 1.

The proposition states that the decentralized decisions approach the centralized decisions as the price-elasticity index or the retailer’s cost share, or both, increase. In the following paragraphs we explore how the decentralized channel profit compares with the decentralized profit as each of the two parameters changes. We also characterize various limiting behaviors, and derive an important lower bound for the performance of the decentralized channel. To these ends, by substituting (10), (11), and (16) into (9), we can show that the manufacturer’s profit in the decentralized channel, denoted by \( \Pi^*_{d,M} \), is given by

\[
\Pi^*_{d,M} = ac \left( \frac{b-1}{b-\alpha} \right)^b (p_c^*)^{-b}z_c^*.
\]

Since \( p_d^* = [(b-\alpha)/(b-1)]p_c^* \) from (17) and \( z_d^* = z_c^* \), we have

\[
\Pi^*_{d,M} = (1 - \alpha) \left( \frac{b-1}{b-\alpha} \right)^b \cdot ac \left( \frac{b-1}{b-\alpha} \right)^b (p_c^*)^{-b}z_c^* = (1 - \alpha) \left( \frac{b-1}{b-\alpha} \right)^b \cdot \Pi^*_{c}, \tag{19}
\]

where the last equality follows from (8), and \( \Pi^*_{c} \) is the centralized channel profit.

Similarly, by substituting (10), (11), and (16) into (13), we can derive the retailer’s profit, denoted by \( \Pi^*_{d,R} \), as

\[
\Pi^*_{d,R} = \left( \frac{b-1}{b-\alpha} \right)^{b-1} \cdot \Pi^*_{c}. \tag{20}
\]

Thus, total decentralized channel profit, denoted by \( \Pi^*_{d} \), is

\[
\Pi^*_{d} = \Pi^*_{d,M} + \Pi^*_{d,R} = \left( \frac{b-1}{b-\alpha} \right)^b (2-\alpha) - 1 \cdot \Pi^*_{c}. \tag{21}
\]

Let \( \Delta_p \) denote the percentage profit loss of the decentralized channel, compared with that of a centralized channel. We have from (21) that

\[
\Delta_p = \frac{\Pi^*_{c} - \Pi^*_{d}}{\Pi^*_{c}} \cdot 100\% = \left( 1 - \left( \frac{b-1}{b-\alpha} \right)^b \frac{(2-\alpha) - 1}{b-1} \right) \cdot 100\%. \tag{22}
\]

**Proposition 5.** With the iso-price-elastic and multiplicative demand model:

1. For any \( b > 1 \), \( \Delta_p \) is decreasing in \( \alpha \), and \( \lim_{\alpha \to 1} \Delta_p = 0 \).

2. For any \( 0 < \alpha < 1 \), \( \lim_{\alpha \to 1} \Delta_p = 0 \), and \( \lim_{\alpha \to \infty} \Delta_p = 1 - (2-\alpha)e^{-(1-\alpha)} \).

3. When \( \alpha = 0 \), \( \Delta_p \) is increasing in \( b \), and \( \lim_{\alpha \to \infty} \Delta_p = 1 - 2/e \approx 26.4\% \).

Figure 1 illustrates the behavior of the channel profit loss function \( \Delta_p \). Part (1) of Proposition 5 implies that, as the retailer bears more of the total channel cost, decentralized channel performance improves. In particular, when the retailer bears almost all the channel cost (i.e., as \( \alpha \to 1 \)), a decentralized channel performs just like a centralized one. The effect of the price-elasticity index \( b \) on \( \Delta_p \) is more complicated. First, we note that when the price-elasticity index is low (i.e., as \( b \to 1 \)), the performance of a decentralized channel approaches that of a centralized channel (i.e., \( \Delta_p \to 0 \)). Second, when the price-elasticity index is very high (i.e., as \( b \to \infty \)), the decentralized channel profit loss approaches a limit that depends on, and decreases in, the retailer’s cost share \( \alpha \). Third, the overall behavior of the channel profit loss function \( \Delta_p \), as the price-elasticity index \( b \) increases from 1 to \( \infty \), also depends on the retailer’s cost share \( \alpha \). Specifically, when \( \alpha = 0 \), \( \Delta_p \) first increases and then decreases slightly in approaching the limit of \( 1 - 2/e \). When \( \alpha \) is relatively large, however, \( \Delta_p \) first increases and then decreases slightly in approaching the limit of \( 1 - (2-\alpha)e^{-(1-\alpha)} \), as seen from Figure 1.

Substituting \( \alpha = 0 \) into (22), we get an upper bound on the channel loss function \( \Delta_p \) for any given \( b \), due to Part (1) of Proposition 5. In conjunction with Part (3), we have that the 26.4% profit loss (i.e., \( 1 - 2/e \)) actually provides a global lower bound on the decentralized channel performance. We summarize this important property in the following.

**Corollary 2.** With the iso-price-elastic and multiplicative demand model, the profit loss of a decentralized channel under consignment with revenue sharing can never exceed 26.4% of a centralized channel.
We next look at how the decentralized channel profit is distributed between the retailer and the manufacturer. From (20) and (21), it follows that the retailer’s share of the channel profit can be calculated as

$$\beta \equiv \frac{\Pi^{*}_{a,R}}{\Pi^{*}_{d,M} + \Pi^{*}_{d,R}} = \frac{b - \alpha}{(2 - \alpha)b - 1}. \quad (23)$$

**Proposition 6.** With the iso-price-elastic and multiplicative demand model, the retailer’s profit share $\beta$

1. is increasing in $\alpha$ for any given $b$, and decreasing in $b$ for any given $\alpha$;
2. approaches 100% either when $\alpha \to 1$ or when $b \to 1$;
3. approaches $1/(2 - \alpha)$ as $b \to \infty$, and can never be below 50%.

Figure 2 illustrates the behavior of the retailer’s profit share $\beta$. Part (3) of Proposition 6 indicates that under a consignment contract with revenue sharing and with the iso-price-elastic demand model, the retailer can *always* extract more than 50% of the total channel profit, even if she does not incur any portion of the channel cost. Furthermore, in conjunction with Proposition 5, we can see that the retailer can actually extract almost all (100%) of the channel profit that nearly equals the profit of a centralized channel under either one or both of the following two circumstances: (1) when she bears nearly all of the channel cost (i.e., $\alpha \to 1$), and (2) when the price-elasticity index is very low (i.e., as $b \to 1$). So, under either or both of these circumstances, a consignment contract is both extremely efficient and desirable from the retailer’s point of view. Of the two, the low price-elasticity situation is more significant to the retailer in the sense that she can extract all of the first-best channel profit without incurring any cost.

5. **Decentralized Channel with Linear Demand Models**

In §4, we completely characterized the decisions and performance of a decentralized channel under the consignment contract with revenue sharing. Specifically, we derived the equilibrium solutions in closed form, which allowed us to characterize analytical properties of, and gain insights into, the performance of a decentralized channel. However, these properties and insights were based on a specific demand function, namely, an iso-price-elastic demand curve multiplied by a random factor. It is thus both interesting and important to know if, or to what degree, those properties and insights may hold in general.

To that end, we consider the following linear and additive demand model,

$$D(p) = a - bp + \epsilon,$$

where $a, b > 0$ are two constants and $\epsilon$ is a random variable with distribution having support on the range $[A, B]$. We assume $a - bc + A \geq 0$ to ensure that the demand is nonnegative when the product is priced at cost.

The above linear and additive demand function is quite different from the iso-price-elasticity and multiplicative model we studied earlier. For example, it does not preserve the iso-price-elasticity property. Its price-elasticity index of the (expected) demand is given by $bp/(a - bp + \mu)$, where $\mu$ is the mean value of the random variable $\epsilon$. The parameter $b$ here is an indicator of the price “sensitivity” of demand and is closely related to the price-elasticity index. Specifically, the price-elasticity index is increasing in $b$ at any price $p$. So, one can consider the parameter $b$ as a surrogate of the price-elasticity index.

It turns out that in the context of our decentralized channel under consignment contract with revenue sharing this linear demand model is much more difficult to analyze than the iso-price-elastic one. In §5.1, we characterize some basic properties of the decentralized decisions and demonstrate the uniqueness of the equilibrium solution. However, unlike the case of the iso-price-elastic demand model, it seems to
be impossible to derive closed-form solutions for the retailer’s decision and for the channel performance measures. In §5.2, we thus use numerical examples to study channel performance for this linear and additive model and, for the sake of curiosity, for a linear and multiplicative model, namely, \( D(p) = (a - bp) \cdot e \). Our numerical studies suggest that the properties and managerial insights generated analytically for the iso-price-elastic model may hold in general for the two linear models.

5.1. Characterization of the Decentralized Decisions for the \( a - bp + e \) Model

Following Petruzzi and Dada (1999, 2001), we define a stocking factor as \( z \equiv q - (a - bp) \). (Notice that, mathematically, the stocking factor defined here for the linear and additive demand model is very different from that for the iso-price-elastic and multiplicative model considered in §§3–4.) The expected profit of the manufacturer and that of the retailer can thus be written as

\[
\Pi_{d,M}(p, z) = -(1 - \alpha)cz + (1 - r)pE[min[q, D]]
\]

\[
= (1 - r)p[a - bp + z - \Lambda(z)]
\]

\[
- (1 - \alpha)c(a - bp + z)
\]  

(24)

and

\[
\Pi_{d,E}(r) = -\alpha cz + rpE[min[q, D]]
\]

\[
= rp[a - bp + z - \Lambda(z)] - \alpha c(a - bp + z),
\]  

(25)

respectively. When specializing on \( r = \alpha = 0 \), the manufacturer’s profit of (24) becomes the centralized channel profit.

Manufacturer’s Problem. For a given revenue share \( r \) chosen by the retailer, the manufacturer’s problem of finding the optimal \((p_d, z_d)\) can be solved sequentially by first finding the optimal \( p_d \) for any given \( z \), and then optimizing \( \Pi_{d,M} \) over \( z \) to find \( z_d \). It is straightforward to verify that \( \Pi_{d,M} \) is concave in \( p \).

As a consequence, the first-order condition uniquely determines \( p_d \) at any \( z \) as

\[
p_d(z) = \frac{(1 - \alpha)c}{2(1 - r)} + \frac{a + z - \Lambda(z)}{2b}.
\]  

(26)

Substituting \( p_d(z) \) into (24), the manufacturer’s profit function then reduces to

\[
\Pi_{d,M}(z) = \frac{1 - r}{4b}[a + z - \Lambda(z)]^2
\]

\[
- \frac{(1 - \alpha)c}{2}[a + z + \Lambda(z)] + \frac{bc^2(1 - \alpha)^2}{4(1 - r)}.
\]  

(27)

The optimal \( z_d \) satisfies the first-order condition of

\[
\frac{d\Pi_{d,M}(z)}{dz} = \frac{1 - r}{2b}[a + z_d - \Lambda(z_d)]\tilde{F}(z_d)
\]

\[
- \frac{(1 - \alpha)c}{2}[1 + F(z_d)] = 0.
\]  

(28)

If the retailer were to choose \( r = 1 - (1 - \alpha)bc/(a + A) \), one can verify from (26) and (28) that the unique optimal response for the manufacturer is to set \( z_d = A, \quad p_d = (a + A)/b \) and, hence, \( q_d = z_d + a - bp_d = 0 \), which results in a zero profit for the manufacturer and for the whole channel. Since the manufacturer’s (optimal) profit is obviously nonincreasing in \( r \), the retailer must choose a value of \( r \) no larger than \( 1 - (1 - \alpha)bc/(a + A) \) in order to induce the manufacturer to produce a positive quantity. We summarize this property of the decentralized channel as the following lemma.

**Lemma 1.** In a decentralized channel, the revenue share \( r \) chosen by the retailer cannot be larger than \( 1 - (1 - \alpha)bc/(a + A) \) in order to induce the manufacturer to produce any positive quantity.

As before, we define \( h(z) = f(z)/\tilde{F}(z) \) as the failure rate function of the probability distribution of \( e \).

We have Theorem 3 with regard to the uniqueness of manufacturer’s optimal solution \((p_d, z_d)\).

**Theorem 3.** If \( 2h(z)^2 + h'(z) \geq 0 \) for all \( r \) such that \( 0 \leq r \leq 1 - (1 - \alpha)bc/(a + A) \),

1. \((p_d, z_d)\) is unique;

2. \[
\frac{d^2\Pi_{d,M}(z)}{dz^2} = \frac{1 - r}{2b}\tilde{F}(z)^2 - \frac{1 - r}{2b}[a + z - \Lambda(z)]f(z)
\]

\[
- \frac{(1 - \alpha)c}{2}f(z) < 0 \quad \text{at } z = z_d.
\]  

(29)

A proof of Theorem 3 can be carried out by following a procedure similar to that of Theorem 2 in Petruzzi and Dada (1999), and we thus omit the details. In our later analyses, the result in Part (2) of Theorem 3 will be used together with those in Proposition 7.

**Proposition 7.**

1. A decentralized channel reaches the same price and production quantity decision as a centralized channel, i.e., \((p_d, q_d) = (p^*_d, q^*_d)\), if and only if \( r = \alpha \).

2. Under the condition \( 2h(z)^2 + h'(z) > 0 \), \( z_d(r) \) is decreasing in \( r \).

3. Under the condition \( 2h(z)^2 + h'(z) > 0 \), \( q_d(r) \) is decreasing in \( r \).

4. Under the conditions \( h'(z) > 0 \) and \( h(A) > 1/(a + A) \), \( p_d(r) \) is increasing in \( r \).

The sufficient condition of \( h(A) > 1/(a + A) \) for the increase of \( p_d(r) \) in \( r \) requires that the failure rate of the demand distribution be sufficiently high at the lower end of its support. Unfortunately, we could not provide an economic interpretation for such a condition.
**Retailer’s Problem.** For a given $r$, the manufacturer’s optimal decision $(p_d, z_d)$ satisfies (26) and (28). Then, substituting (26) and (28) into (25), we can rewrite the retailer’s expected profit function as

\[
\Pi_{d, R}(r) = \frac{r}{4b} \left[ a + z_d - \Lambda(z_d) \right]^2 - \frac{br(1 - \alpha)^2 c^2}{4(1 - r)^2} - \alpha c \left[ \frac{a + z_d + \Lambda(z_d)}{2} - \frac{bc(1 - \alpha)}{2(1 - r)} \right].
\]

(30)

After some algebra, we derive the first-order derivative of $\Pi_{d, R}(r)$ as

\[
\frac{d\Pi_{d, R}(r)}{dr} = \frac{bc^2(1 - \alpha)}{4(1 - r)^2} \left\{ (1 - \alpha) \left[ \frac{1 + F(z_d)}{1 - F(z_d)} \right]^2 + 2\alpha \right. \\
+ \left. \frac{2[1 + F(z_d)]^2 (\alpha - r)}{I(r, z_d)} - (1 - \alpha) \frac{1 + r}{1 - r} \right\}.
\]

(31)

where

\[
I(r, z_d) \equiv (1 - \alpha)bcF(z_d) + (1 - r)[a + z_d - \Lambda(z_d)]f(z_d) - (1 - r)\bar{f}(z_d) \\
= -2b \frac{d^2\Pi_{d, M}(z_d)}{dz^2} > 0,
\]

(32)

where the inequality follows from (29).

**Theorem 4.** If $F(\cdot)$ has increasing failure rate, i.e., $h'(z) \geq 0$, then the retailer’s profit function $\Pi_{d, R}(r)$ is unimodal on $[0, 1 - (1 - \alpha)bc/(a + A)]$ and has a unique maximizer $r^*$ satisfying $\alpha < r^* < 1 - (1 - \alpha)bc/(a + A)$.

In conjunction with Parts (3) and (4) of Proposition 7, Theorem 4 leads to the following conclusion about the decentralized decision and channel profit.

**Corollary 3.** Under the conditions of $h'(z) > 0$ and $h(A) > 1/(a + A)$, the retail price in a decentralized channel is always higher than the centralized price, and the production quantity is always lower. As a consequence, the profit of a decentralized channel is always lower than the profit of a centralized channel.

While our analyses requires the sufficient condition $h(A) > 1/(a + A)$, in general it seems highly intuitive that the decentralized system will employ a higher price and a lower quantity due to double marginalization. It remains to be explored as to how to relax such a sufficient condition or to come up with conditions such that the conclusions of Corollary 3 are not true.

Regarding the distribution of the decentralized channel profit $\Pi_d^*$ between the retailer and the manufacturer, we know from Proposition 7 that if the retailer were to choose $r = \alpha$, the decentralized channel profit would equal the centralized channel profit, i.e., $\Pi_d^*(\alpha) = \Pi_c^*$. In that case, we can show that the retailer’s profit is $\Pi_{d, R}(\alpha) = a\Pi_c^*$. Theorem 4 implies that the retailer’s profit is maximized by $r = r^* > \alpha$. That is, $\Pi_{d, R}(r^*) > \Pi_{d, R}(\alpha) = a\Pi_c^*$. Furthermore, we know from Corollary 3 that $\Pi_c(r^*) < \Pi_c^*$. Thus, we have $\beta = \Pi_{d, R}(r^*)/\Pi_c(r^*) > a\Pi_c^*/\Pi_c^* = \alpha$. That is, in a decentralized channel, the retailer always captures a profit share that is strictly bigger than her share of the channel cost. (Note, the argument and conclusion here apply also to the iso-price-elastic model of §3. However, we were able to reach sharper conclusions analytically for that model, e.g., Equation (23) and Proposition 6.)

### 5.2. Numerical Comparisons

We performed numerical studies of channel performance under the linear and additive model analyzed above and, for the sake of curiosity, under the linear and multiplicative model $D(p) = (a - bp) \cdot \varepsilon$. For each of the two models, we considered uniform and exponential probability distributions for the random variable $\varepsilon$. Specifically, we tested the uniform distributions on $[0, B]$ for $B = 5, 10, 15, \ldots, 100$, and the exponential distributions with means $\mu = 5, 10, 15, \ldots, 50$. We normalized the cost parameters $c = 1$ and considered $a = 5, 10, 15, \ldots, 50$. We found that the performance curves, across all the combinations of demand models, distributions, and parameters, were strikingly similar. Therefore, in the following we illustrate the channel performance of a single test example and compare the results with that of the iso-price-elastic model.

Figures 3 and 4 show the decentralized channel profit loss function and the retailer’s share of channel profit, respectively, generated using the linear and additive model with $a = 30$, $c = 1$, and $\varepsilon$ being exponentially distributed with the mean value $\mu = 10$. Comparing Figures 3 and 4 with Figures 1 and 2, respectively, we observe that while some minor differences exist, the overall trends of the two sets of...
price sensitivity

namely, the numerical examples. (Proposition 4). Here we omit the detailed data from
the retailer's optimal revenue share

increases, and improves as the retailer incurs more
of the channel cost. With the iso-price-elastic and
multiplicative demand model, the decentralized channel profit; however, usually cannot be below 73.6%
(=100%−26.4%) of the centralized channel profit.
The profit share extracted by the retailer in the channel
decreases with the price elasticity and increases
with the retailer’s cost share; however, it cannot be
lower than 50%. When the price elasticity is low, or
when the retailer’s cost share approaches 100%, or
both, the retailer can extract almost all of the decen-
tralized channel profit, and that profit is nearly equal
to the centralized channel profit. We also analyzed a
linear demand model and demonstrated numerically
that the key properties and insights obtained analyti-
cally for the iso-price-elastic model hold strongly for
the cases we examined.

This research is a first step toward studying con-
signment contracts with revenue sharing. There are
other intuitively intriguing and broader questions to
ask. For example, why would a retailer let her sup-
pliers choose the selling price and delivery quantity?
Under what general settings would such a contract
structure emerge as being more attractive than other
contract forms? For example, without information
asymmetry, a “truly powerful” retailer could extract
all the maximum channel rent by simply paying the
manufacturer his production cost (or his cost plus e),
ordering the centralized quantity and setting the cen-
tralized retail price. In the following paragraph we
offer some limited and speculative discussion from a
practical point of view.

From industry examples (e.g., arts galleries, antique
stores, Amazon.com’s marketplace, etc.), we observe
that a common feature of retailers using such a con-
signment contract with revenue sharing is that they
all face a large number of suppliers who may sell
different products in relatively small quantities.
From a retailer’s point of view, an obvious advan-
tage of allowing suppliers to make price and quantity
decisions is that it relieves her from activities such
as checking product quality, appraising the value of
a product, forecasting demand, etc. With the number
of suppliers and products being large, these activities
can be costly to perform and the retailer may simply
not have the expertise or capacity required to perform
them. That said, a consignment contract with revenue
sharing may not necessarily be cheaper to administer
and enforce than, say, a simple wholesale price con-
tract. Proper procedures and measures to enforce a
retail price and to verify sales are needed to imple-
ment such a contract. Using the models in this paper,
one can easily show that, after receiving the deliv-
ered quantity from the supplier, a retailer always has
incentive to sell them at a price higher than the price
chosen by the supplier, and such an action will surely
hurt the supplier. Second, if it is possible to “cheat”
figures, especially their limiting behaviors, are strik-
ingsly similar. This indicates that the key properties of
Propositions 4 and 5 obtained analytically for the iso-
price-elastic model may hold for the linear models.

Our numerical examples suggest that other ana-
tycal properties of §4 regarding decentralized deci-
sions also hold for the linear models. For example,
the retailer’s optimal revenue share \( r^* \) is increasing
in the retailer’s cost share \( \alpha \) and is decreasing in
price sensitivity \( b \) (Proposition 3); deviations of the
decentralized equilibrium price and production quantity
from the centralized optimal price and quantity,
namely, \( \Delta_p \) and \( |\Delta_q| \), respectively, are decreasing both
in the retailer’s cost share \( \alpha \) and in price sensitivity \( b \)
(Proposition 4). Here we omit the detailed data from
the numerical examples.

6. Summary and Future Research Directions

We studied the consignment contract with revenue
sharing where a retailer offers a revenue share scheme
between herself and her supplier, and the supplier
then chooses the delivery quantity and retail price
of his product. In such a channel, firms consider
solely their individual profits in choosing an action(s)
to take. Individual firms’ profits, however, are not
aligned with the channel profit and, hence, the chan-
nel cannot achieve its best (centralized) performance.
The key questions of managerial interest then become:
How well does such a decentralized channel perform?
and How well do individual firms perform?

Using an iso-price-elastic and multiplicative de-
mand model, we fully characterized the decentral-
ized decisions and derived closed-form performance
measures. Those elegant analytical results allowed
us to gain managerial insights into this contract.
Specifically, we showed that the performance of a
decentralized channel degrades as price elasticity
increases, and improves as the retailer incurs more
of the channel cost. With the iso-price-elastic and

<table>
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<th>( \alpha )</th>
<th>( \beta )</th>
</tr>
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<tr>
<td>0</td>
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</tr>
<tr>
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</tr>
<tr>
<td>0.5</td>
<td>80%</td>
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<tr>
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<td>70%</td>
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</tbody>
</table>

Figure 4 Retailer’s Profit Share \( \beta \) for a Linear Demand Model

\[ \alpha \rightarrow 1 \]

\( \alpha = 0 \)

\( \alpha = 0.25 \)

\( \alpha = 0.5 \)

\( \alpha = 0.75 \)

\( \alpha = 1 \)

Price sensitivity \( b \)
on sales, a retailer would surely do so; see Gerchak and Khmelnitsky (2002) for real-world examples.

The above discussions also lead to future research directions. One is to extend the model to a situation where the retailer faces multiple suppliers, and where the products of different suppliers can be either complementary or substitutable (e.g., Netessine and Zhang 2002). Another extension is to consider cases of asymmetric information (e.g., Corbett and de Groote 2000, Corbett and Tang 1998, Ha 2001, Deshpande and Schwarz 2002). For example, the supplier may have more accurate demand forecast than the retailer, the two firms may not have full information about each other’s cost, etc.

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Appendix. Mathematical Proofs

Proof of Theorem 1. First, for any fixed z with A ≤ z ≤ B, it follows from (4) that δΠ(p, z)/δp = ap−(b+1)[bz−(b−1)[z − Λ(z)]]. Since ap−(b+1) > 0, δΠ(p, z)/δp > 0 implies p∗(z) = bc/[z − Λ(z)], which is (6). p∗(z) is the unique maximizer of Π(p, z), since δΠ(p, z)/δp > 0 for all p < p∗(z) and δΠ(p, z)/δp < 0 for all p > p∗(z). It can be shown that p∗(A) = bcA/[A − Λ(A)] = bc/(b−1) > c > 0, and that p∗(z) is increasing in z.

We next find the maximizer z∗ of Π,[P∗(z), z]. By the chain rule, we have

\[
\frac{d\Pi}{dz} = \frac{\delta \Pi[p^*_z(z), z]}{\delta p} + \frac{\delta \Pi[p^*_z(z), z]}{\delta p^*_z(z)} \frac{dp^*_z(z)}{dz} = a[p^*_z(z)]^{-b} \frac{[1 − F(z)] − c}{[z − Λ(z)]} \cdot G(z) \quad \text{with}
\]

where we have used the fact that δΠ[p∗(z), z]/δp = 0 due to the optimality of p∗(z). Since the first factor in the above expression is always positive, first-order condition requires that the optimal z∗ satisfy G(z) = 0, which gives us (7). Such a z∗ always exists in the support interval (A, B) of F(), since (1) G(z) is continuous, and (2) G(A) = A > 0 and G(B) = −(b−1)ζ < 0. To verify the uniqueness of z∗, we have G(z) = 1 − F(z)[1 −bz(z)] and G(z) = −h(z)G(z) = −[1 − F(z)][h(z) + zζ(z)]. Now, if h(z) + zζ(z) > 0, then G(z) < 0 at G(z) = 0, which implies that G(z) itself is a unimodal function. This, in conjunction with G(A) = A > 0 and G(B) = −(b−1)ζ < 0, guarantees the uniqueness of z* and A < z* < B. We thus complete the proof of Theorem 1.

Proof of Proposition 1. Part (1) From (7), we have that z∗ solves G(z∗) ≡ z∗ + (b−1)Λ(z∗) − bζF(z∗) = 0. By the implicit function rule, we have dz∗/db = (L(z∗)/∂G(z∗)/∂ζ), where L(z∗) = z∗F(z∗) − Λ(z∗). Now, L(z∗) > 0, since L(A) = A > 0 and L(z∗) = z∗F(z∗) > 0. ∂G(z∗)/∂ζ < 0, since G(A) = A > 0, G(B) = −(b−1)ζ < 0, and z∗ solving G(z∗) = 0 is unique (Theorem 1). Thus, dz∗/db < 0, that is, z∗ is decreasing in b. z∗ is not affected by c, since (7) does not involve c.

Part (2) Since z∗ is not affected by c, we see from (6) that p∗ = p∗(z∗) is linearly increasing in c. With respect to b, we have dp∗/db = dp∗/db + (dp∗/∂ζ)(dz∗/db). Now,

\[
\frac{dp^*_z}{db} = \frac{cζ^∗}{z^∗ − Λ(z^∗)} (b − 0) < 0 \quad \text{and}
\]

\[
\frac{dp^*_z}{∂ζ} = \frac{bc}{b − 1} \frac{L(z^∗)}{[z^∗ − Λ(z^∗)]^2} > 0.
\]

In conjunction with dz∗/db < 0, we see that dp∗/db < 0. This completes the proof of Proposition 1. □

Proof of Proposition 2. Part (1) z∗ is determined by Equation (11), which does not have r, c, or ζ as parameters. Furthermore, Equation (11) is identical to Equation (7) from which z∗ is determined.

Part (2) The result follows from a simple comparison of (10) with (6) and the fact that z∗ = z∗. □

Proof of Theorem 2. It follows from (15) that g′(r) = (1 − r−3/2)[(1 − a)r + (a−1)(1 − br) + a(b−1)](1 − r)−1−r. We can check that g′(r) > 0 as r → 0+, and g′(r) < 0 as r → 1−. This implies that g(r) reaches its maximum at some interior point on [0, 1]. Solving g′(r) = 0, we get the unique maximizer r∗ as given in (16).

Proof of Proposition 3. Part (1) Otherwise, r∗ = (a(b−2)−1)/(b−α) < α implies (α−1)−1 < 0, which is not true. Part (2) It follows directly from the facts that with 0 < α < 1 and b > 1,

\[
\frac{dr^∗}{dα} = \left(\frac{b − 1}{b − α}\right)^2 > 0 \quad \text{and} \quad \frac{dr^∗}{db} = 0 < 0
\]

Proof of Proposition 4. Part (1) The conclusions are obvious.

Part (2) We can write |Δa| = 1−L(b), where L(b) = [(b−1)/(b−α)]α. To show that |Δa| is decreasing in b, we just need to show that L(b) is increasing in b, or equivalently that L(b) > 0 for b > 1. Now, L′(b) = L(b) · l(b), where l(b) = (1−a)b/[b(b−1)(b−α)] + log[(b−1)/(b−α)]. The first factor in L(b) is positive. The following argument shows that the second factor, namely, l(b), is also positive and, hence, L(b) is always positive. For l(b), we have l′(b) = (1−a)α[b−1]/[b−1(b−α)]² < 0 for b > 1, and, limb→1 l(b) = 0. Thus, l(b) > 0 for b > 1. This completes the proof of |Δa| decreasing in b. The limiting behavior of |Δa| with respect to b and α can be demonstrated by following some straightforward calculus steps. □

Proof of Proposition 5. Let u(a, b) ≡ [(b−1)/(b−α)]α. [(2−a)b−1]/(b−1). Part (1) From (22), to show Δa is decreasing in α, we only need to show that u(a, b) is increasing in α, or, equivalently, that du(a, b)/dα > 0. After some algebra, we have du(a, b)/dα = (a−(b>a)−1)/(b−a)−1 > 0, lima→1 Δa = 0 can be verified by simply substituting α = 1 into (22).
Part (2) We have
\[
\lim_{b \to 1} \Delta_{II} = 1 - \lim_{b \to 1} \frac{[(2 - \alpha)b - 1]}{(b - \alpha)^b} = 1 - \lim_{b \to 1} e^{(b - \alpha) \log(b - 1) - (b - \alpha) - (2 - \alpha)b + 1/(b - \alpha)^b} = 1 - e^{(1 - \alpha)(2 - \alpha)}.
\]

Then, we can show that
\[
\frac{dp(r)}{dr} = \frac{dF(z_r)}{dz} \frac{dz_r}{dr} = \frac{\tilde{F}(z_r)}{b(1 + F(z_r))^2} g(z_r) \frac{dz_r}{dr},
\]
where \(g(z_r) = 1 + F(z_r) - [a + z_r - \Lambda(z_r)] h(z_r).

Part (3) When \(a = 0\), \(k(b) \equiv u(b, c) = [(b - 1)/b]^2 [(2b - 1)/(b - 1)^2] \). From (22), to show that \(\Delta_{II}\) is increasing in \(b\), we only need to show that \(a(b)\) is decreasing in \(b\), or, equivalently, that \(k'(b) > 0\). After some algebraic manipulations we have \(k'(b) = (b - 1)^2 1/(b - 1) \log(b - 1)/b\), where \(v(b) = (2b - 1) \log(b - 1)/b\). Now, it can be shown that \(\lim_{b \to 1} v(b) = 2\).

Part (2) From (23), we have \(s(b, \alpha) = (b - 1)/(b - \alpha)^2\), which means that \(s\) is increasing in \(\alpha\), and \(s(\alpha, b) = (a - \alpha)/[(2 - \alpha)b - 1] < 0\), which indicates that \(s\) is decreasing in \(b\).

Part (3) From (23), we have \(s(\alpha, b) = 1/(2 - \alpha)\). Since \(b\) is increasing in \(\alpha\), this lower bound for \(\alpha\) can be increased by letting \(\alpha \to 0\) and \(b \to \infty\). We have from (23) that \(\lim_{b \to \infty, \alpha \to 0} s(\alpha, b) = (a - \alpha)\).

Proof of Proposition 7. Part (1) Since the production quantity is uniquely determined by \((p, z)_t\), we only need to show that \((p_t, z_t) = (p^*_t, z^*_t)\). When \(r = \alpha = 0\), the manufacturer’s problem in (24) reduces to the centralized channel problem. Thus, specializing on \(r = \alpha = 0\), Equations (26) and (28) provide the optimal conditions for the decentralized decision \((p^*_t, z^*_t)\). One can verify that when \(r = \alpha\), the optimal conditions of (26) and (28) for the decentralized decision \((p_t, z_t)\) reduce to those of the centralized decision \((p^*_t, z^*_t)\). Furthermore, we know from Theorem 3 that the solution to (26) and (28) is unique. As a consequence, we must have \((p_t, z_t) = (p^*_t, z^*_t)\).

Part (2) \(z_t\) as a function of \(r\) is determined through (28), from which we can show that
\[
\frac{dz_t(r)}{dr} = \frac{[a + z_t - \Lambda(z_t)] \tilde{F}(z_t)/2b}{d^2 \Pi_{A,M}(z_t)/dz^2_t} < 0,
\]
where the inequality follows from Part (2) of Theorem 3. Thus, \(z_t\) is decreasing in \(r\).

Part (3) Using (26), we get
\[
q_t(r) = a - bp_t(r) + z_t(r) = \frac{a + z_t + \Lambda(z_t)}{2} - \frac{bc(1 - \alpha)}{2(1 - r)}.
\]
of $d\Pi_{d,k}(r)/dr$ is determined by $[X(r, z_d) - Y(r)]$. We also know that $d\Pi_{d,k}(r)/dr > 0$ at $r = \alpha$ and $d\Pi_{d,k}(r)/dr < 0$ at $r = 1 - (1 - \alpha)bc/(a + A)$. As a consequence, to demonstrate that $d\Pi_{d,k}(r)/dr$ changes sign only once, we only need to show that $[X(r, z_d) - Y(r)]$ is decreasing in $r$, which is guaranteed if $X(r, z_d)$ is decreasing and $Y(r)$ is increasing. It is obvious that $Y(r)$ is increasing. Below we show that $X(r, z_d)$ is decreasing in $r$.

Since $z_d$ is uniquely determined by and decreasing in $r$ (Part 2 of Proposition 7), to show that $X(r, z_d)$ is decreasing in $r$ is equivalent to showing that it is increasing in $z_d$, or $dX(r, z_d)/dz = \partial X(r, z_d)/\partial z + [\partial X(r, z_d)/\partial r] [dr(z_d)/dz] > 0$. After some algebra we have

$$
\frac{dX(r, z_d)}{dz} = (1 - r) \frac{f(z_d)}{h(z_d)} + \frac{(1 - r) \bar{F}(z_d)^2}{h(z_d)[a + z_d - \Lambda(z_d)]}
+ K(r, z_d) + M(z_d),
$$

where

$$K(r, z_d) = (1 - r) \left\{ [a + z_d - \Lambda(z_d)] \frac{\bar{f}(z_d)^2}{2[1 + F(z_d)]} \right\}$$

and

$$M(z_d) = bc(1 - \alpha) \left\{ \frac{\bar{f}(z_d)^2}{2[a + z_d - \Lambda(z_d)]} \right\}.$$

The first two terms in $dX(r, z_d)/dz$ are each positive. $M(z_d)$ is positive, since it is increasing in $z_d$ and $M(A) = bc(1 - \alpha) \cdot [f(A) - 1/[2(a + A)] > 0$, where the inequality is implied by Part 2 of Theorem 3, specializing on $r = 1 - (1 - \alpha)bc/(a + A)$ and $z = z_d = A$. Using a similar argument, we can show that $K(r, z_d)$ is also positive. We thus complete the proof of Theorem 4. □

References


Deshpande, V., L. Schwarz. 2002. Optimal capacity choice and allocation in centralized supply chains. Working paper, Purdue University, West Lafayette, IN.


