Channel Coordination Under Price Protection, Midlife Returns, and End-of-life Returns in Dynamic Markets

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1. Introduction

**Background:** Under short product life cycles and high demand uncertainty environment, suppliers may extend channel policies that expand the terms of trade beyond the wholesale price.

**Problem:** can the combination guarantee channel coordination? can the combination assure win-win?

**What this paper do:**

Exploring how simultaneous channel policies can be used to improve supply chain performance in dynamic markets.

Exploring the role of various channel policy combinations when firms can dispose of product during and at the end of the life cycle.
2. Literature survey

✓ Lee et al. (2000) explore the use of price protection with a two-period model, ignoring the possibility of returns or the disposal of unsold inventory at midlife.

✓ Lariviere and Porteus (1999) explore a price-only contract in a newsvendor setting.

✓ Pasternack (1985) explores the role of returns in the context of perishable commodities, i.e., products for which there is a single buying opportunity at the start of the life cycle and a single return opportunity at the end of life.

Tsay (1999) and Brown and Lee (1998) show, respectively, that quantity flexibility and options arrangements can be instruments to achieve channel coordination in a one-demand occurrence environment.

Barnes-Schuster et al. (1998) study the role of options in a two-demand occurrence environment where demand is correlated.

Eppen and Iyer (1997) and Milner and Rosenblatt (1997) each use two-demand occurrence models to focus on the retailer's behavior under, respectively, backup agreements and per-unit penalty arrangements.
3. The model

- Notation:
  - $P_i$: retail price in period $i=1,2$
  - $W_i$: wholesale price in period $i=1,2$
  - $C_i$: marginal cost in period $i=1,2$
  - $V_i$: salvage value disposed
  - $B_i$: returning rebate
  - $\beta$: price protection magnitude parameter, $\beta \in [0,1]$
  - $\xi_i$: random variable denoting the demand in period $i$ having density $\phi_i(\cdot)$, $\mu_i$

Distribution and mean
Assumption A1. $0 < c_i < w_i < p_i$, $v_i \leq b_i \leq w_i$ for $i = 1, 2$; $v_2 \leq v_1 < c_2 \leq c_1$, $w_2 \leq w_1$, $p_2 \leq p_1$.

Assumption A2. $p_i$, $c_i$, $v_i$, and $\Phi_i(\cdot)$ are exogenous; $w_i$, $b_i$, and $\beta$ are endogenous.

Assumption A3. No lump sum side payment is allowed.

We assume $\phi_i(\xi_i) > 0$ for all $\xi_i \geq 0$; the analysis can be extended to any support $[l, u)$, where $0 \leq l < u \leq \infty$. Further, we assume all retailer orders can be filled (i.e., manufacturer capacity is infinite).
we restrict our attention to situations in which the retail and wholesale prices, manufacturing cost, and salvage value are declining over time.
3.1 The Integrated Channel and the Independent Retailer Under No Channel Policy

✓ $\pi_1 =$ the integrated channel’s total expected profit over the two periods
✓ $\pi_2 =$ the integrated channel’s expected profit in Period 2
✓ given that the integrated channel’s ending stock in Period 1 before salvaging is x

The integrated channel’s problem is described in the following dynamic program:

$$\pi_1 = \max_{y \geq 0} \{ p_1 E \min(y, \xi_1) - c_1 y + E \pi_2((y - \xi_1)^+) \}, \quad (1)$$

$$\pi_2(x) = \max_{y \geq 0} \{ p_2 E \min(y, \xi_2) - c_2 (y - x)^+$$

$$\quad + v_1 (x - y)^+ + v_2 E (y - \xi_2)^+ \}.\$$
Define:

\[
S_2 := \Phi_2^{-1}\left( \frac{p_2 - c_2}{p_2 - v_2} \right) \quad \text{and} \quad \bar{S}_2 := \Phi_2^{-1}\left( \frac{p_2 - v_1}{p_2 - v_2} \right)
\]

✓ The optimal policy in Period 1 is to order up to a quantity, call it $S_1$.
✓ The optimal policy at the start of Period 2 is given by the following:
  - if the stock level is less than $S_2$, then order up to $S_2$;
  - if the stock level is greater than $\bar{S}_2$, then dispose down to $\bar{S}_2$;
  - if the stock level is between these thresholds, then neither order nor dispose.
Consider the setting in which the retailer is independent and the sole terms of trade are the wholesale prices, i.e., no channel policy is used:

The retailer's Period 2 order-up-to quantity is strictly less than that of the integrated channel. The retailer's Period 1 order-up-to quantity in general differs from that of the integrated channel.

Conclusion: when the retailer is independent and no channel policy is employed, the total chain profit is less than that of the integrated channel.
3.2 Independent Retailer Under PEM

✓ Under PEM: Price protection can be exercised when the wholesale price drops; Returns can be exercised both at this time and at the end of the life cycle.

✓ Price protection model:

   For each unit on which the retailer takes price protection, she receives a credit of \((w_1-w_2)\beta\) where \(\beta \in [0,1]\).

   For each unit she returns in period \(i\), she receives a rebate of \(b_i\) \((i = 1,2)\).
We assume:
\[ b_1 < w_2 + (w_1 - w_2) \beta \]
So the retailer under P and M with excess stock at the end of Period 1 is restricted.

The optimal order-up-to quantities:
\[ T_i = \Phi_i^{-1}\left(\frac{p_i - w_i}{p_i - b_i}\right) \quad i = 1, 2. \]

Let \( R_{2x} = \) retailer's expected profit under PEM, given that the retailer's ending stock in Period 1 before returns is \( x \).
The retailer's problem is described in the following dynamic program:

\[
R_1 = \max_{y \geq 0} \left\{ p_1 E \min(y, \xi_1) - w_1 y + E R_2((y-\xi_1)^+) \right\},
\]

\[
R_2(x) = \max_{y \geq 0} \left\{ p_2 E \min(y, \xi_2) - w_2 (y-x)^+ + b_1 (x-y)^+ + (w_1 - w_2) \beta \min(x, y) + b_2 E (y-\xi_2)^+ \right\}.
\]

Define

\[
\overline{T}_2 := \Phi_2^{-1} \left( \frac{p_2 + (w_1 - w_2) \beta - b_1}{p_2 - b_2} \right).
\]
Note that (2) implies \( T_2 < \overline{T}_2 \).

The optimal dispose-down-to quantity is \( \overline{T}_2 \), and from its definition, it is increasing in the price protection credit and decreasing in the midlife return rebate.

**Lemma 1.** The optimal quantity achieved by ordering or returns \( T_2 \) for the retailer in Period 2 under PEM is given by the following: if \( x \leq T_2 \), then \( T_2 = T_2 \); if \( T_2 < x < \overline{T}_2 \), then \( T_2 = x \); if \( x \geq \overline{T}_2 \), then \( T_2 = \overline{T}_2 \).

The policy: If \( x \leq T_2 \), then take the price protection credit on \( x \) units and order \( T_2 - x \) units; if \( T_2 < x < T_2 \), then take the price protection credit on \( x \) units and order none; if \( x \geq T_2 \), then return \( x - T_2 \) units and take the price protection credit on \( T_2 \) units.
Define:
\[ T_0 := \Phi_1^{-1} \left( \frac{p_1 - w_1}{p_1 - w_2 - (w_1 - w_2)\beta} \right). \]

The quantity $T_0$ is the myopic order-up-to quantity in Period 1 when each unit unsold at the end of Period 1 has value $w_2 + (w_1 - w_2)\beta$.

**Lemma 2.** The optimal policy for the retailer in Period 1 under PEM is to order up to $T_1$, which is given by the following: if $T_0 \leq T_2$, then $T_1 = T_0$; if $T_2 < T_0 \leq T_2$, then $T_1$ satisfies

\[
p_1 - w_1 - [p_1 - w_2 - (w_1 - w_2)\beta] \Phi_1(T_1) \\
\quad + (p_2 - w_2) \Phi_1(T_1 - T_2) \\
\quad - (p_2 - b_2) \int_0^{T_1-T_2} \Phi_2(T_1 - \xi_1) d\Phi_1(\xi_1) = 0, \quad (4)
\]
it also holds that $T_2 < T_1 < T_0$; if $T_0 > T_2$ and $k \leq 0$, then $T_1$ satisfies (4) and it also holds that $T_2 < T_1 < T_0$; if $T_0 > T_2$ and $k > 0$, then $T_1$ satisfies:

$$p_1 - w_1 - [p_1 - w_2 - (w_1 - w_2)\beta]\Phi_1(T_1) + (p_2 - w_2) \times \Phi_1(T_1 - T_2) - [p_2 + (w_1 - w_2)\beta - b_1]\Phi_1(T_1 - T_2)$$

$$-(p_2 - b_2) \int_{T_1 - T_2}^{T_1 - T_2} \Phi_2(T_1 - \xi_1)d\Phi_1(\xi_1)=0,$$ \hspace{1cm} (5)

and it also holds that $T_2 < T_1 < T_0$.

$k$ is the first derivative of the retailer's Period 1 objective function with respect to the Period 1 order.
4. Channel coordination under EM

✓ Let \( r \): the retailer’s profit

✓ Let \( m \): the manufacturer’s profit

(in a decentralized no channel policy environment)

✓ Let \( \pi \) be the profit in an integrated channel.

✓ Define: \( \lambda := (p_1 - p_2) \int_0^{S_1} \xi_1 d\Phi_1(\xi_1) \).

**Theorem 1.** Consider the channel policy combination of \( (w_1(\varepsilon), w_2(\varepsilon), b_1(\varepsilon), b_2(\varepsilon)) \) for \( \varepsilon \in (0, p_2 - c_2) \):

Let \( w_1(\varepsilon) = p_1 - (p_1 - p_2)\Phi_1(S_1) - \varepsilon[p_1 - c_1 - (p_1 - p_2)\Phi_1(S_1)]/(p_2 - c_2) \), \( w_2(\varepsilon) = p_2 - \varepsilon \), and \( b_i(\varepsilon) = p_2 - \varepsilon(p_2 - v_i)/(p_2 - c_2) \); \( i = 1, 2 \).
(a) The channel policy combination achieves coordination.

(b) The resulting profit to the manufacturer and retailer is

$$\hat{m}(\varepsilon) = [\pi - \lambda][1 - \varepsilon/(p_2 - c_2)],$$

and

$$\hat{r}(\varepsilon) = [\pi - \lambda]\varepsilon/(p_2 - c_2) + \lambda.$$

(c) If \(m < \pi - \lambda\), then for

$$\varepsilon \in \left(\frac{(p_2 - c_2)(r - \lambda)}{\pi - \lambda}, \frac{(p_2 - c_2)(\pi - m - \lambda)}{\pi - \lambda}\right),$$

\(\hat{m}(\varepsilon) > m\) and \(\hat{r}(\varepsilon) > r\), i.e., win-win is achieved. If \(m \geq \pi - \lambda\), then \(\hat{m}(\varepsilon) < m\), i.e., the manufacturer profit is strictly lower under coordination, and win-win cannot be achieved.
To summarize: the manufacturer must give to the retailer a margin, $pl - w1$, that is bounded away from zero, and the retailer profit is consequently bounded away from zero as well. Essentially, declining retail prices force the channel coordinating EM manufacturer to set a lower wholesale price and consequently garner lower profit.
5. Channel Coordination Under PEM

**Theorem 2.** Consider the channel policy combination of \((w_1(\varepsilon), w_2(\varepsilon), \beta(\varepsilon), b_1(\varepsilon), b_2(\varepsilon))\) for \(\varepsilon \in (0, p_2 - c_2)\): Let \(w_1(\varepsilon) = p_1 - \varepsilon(p_1 - c_1)/(p_2 - c_2)\), \(w_2(\varepsilon) = p_2 - \varepsilon\), \(b_i(\varepsilon) = p_i - \varepsilon(p_i - v_i)/(p_2 - c_2)(i = 1, 2)\), and

\[
\beta(\varepsilon) = \begin{cases} 
(p_1 - p_2)(p_2 - c_2 - \varepsilon) \\
/[(p_1 - p_2)(p_2 - c_2 - \varepsilon) + \varepsilon(c_1 - c_2)] & \text{if } p_1 > p_2, \\
0 & \text{else}.
\end{cases}
\]

(a) The channel policy combination achieves coordination.

(b) The resulting profit to the manufacturer and retailer is \(\tilde{m}(\varepsilon) = \pi[1 - \varepsilon/(p_2 - c_2)]\), and \(\tilde{r}(\varepsilon) = \pi \varepsilon/(p_2 - c_2)\).
 Achieving coordination requires inducing the retailer to (1) order a sufficiently large quantity in Period 1 and (2) carry unsold inventory into Period 2.

To summarize: First, under declining retail prices, the maximum profit the manufacturer can capture under PEM is larger than the maximum profit she can capture under EM.

Second, a properly designed PEM policy guarantees channel coordination and win-win even if prices are decreasing and manufacturer profit under no channel policy is relatively high.
6. Discussion

- why we do not generally see subsidies for carrying inventory, analogous to price protection, in environments with static retail prices: Under static retail prices, the use of returns is sufficient to guarantee coordination and win-win.

- why price protection is used in addition to returns in declining retail price environments: Under declining retail prices, the use of returns is insufficient to guarantee win-win, but the addition of price protection restores this property.

- provide insight into the role of additional degrees of freedom provided by P, M, and E in achieving channel coordination.
7. Further research

✓ Under a increasing price environment, how to achieve channel coordination
✓ Consider backorder
✓ Consider stockout cost
Thanks for your attention!