Collusion and discrimination in organizations

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Abstract

In a principal–multi-agent setting we investigate how optimal contracts should be modified under relative performance evaluation when agents collude. Agents may write side-contracts, which are not contingent on their effort choices but indirectly control them through side-transfers. We show that the optimal collusion-proof contract is to introduce a “discriminatory policy” in the sense that the wage schemes offered to agents depend on their identities even if they are identical with respect to productive abilities. Such discriminatory wage schemes explain the organizational strategy of “divide and conquer” as an optimal response to collusion.

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1. Introduction

It is well known that relative performance evaluation can sometimes resolve the moral hazard problems, by making the compensation of agents contingent on the relative ranking of their performances (for example, consider rank–order tournaments).\(^1\) However, it is also well known that agents have strong incentives to agree on shirking when such a scheme is introduced,\(^2\) because the relative ranking of their performances is not changed when they all shirk compared to when they all work

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\(^1\)See [9,12] for rank–order tournaments.

hard, and they can save their effort costs by shirking. Thus relative performance evaluation is vulnerable to collusion.

In this paper we address the issue of what organizational responses are derived when agents collude under relative performance evaluation in a principal–multi-agent setting. The main purpose of this paper is to show that wage discrimination is introduced by the principal to deter collusion among agents. A discriminatory wage in this context means that the wage schemes offered to agents depend on their identities even if they are identical with respect to their productive abilities. In this sense we show that the optimal contract takes a non-anonymous form: one agent is offered a higher-powered incentive scheme than the other agent even when they are symmetric. The favored agent can also obtain a positive rent, but the unfavored agent cannot.

The intuition for the above result is explained as follows: if collusion does not occur, the agent who is offered a high-powered incentive scheme is well motivated to work hard and obtain a high wage premium. Then he or she would not agree on any side-contract that induces him or her to shirk if a large side-transfer was not obtained on average. In other words the agent would require a high (expected) side-transfer for shirking in exchange for giving up the high wage premium that would be otherwise obtained by working hard. However, the other agent who is offered a low-powered incentive scheme would suffer from paying such a high (expected) side-transfer, because his or her wages are not sensitive to the other agent’s effort (low-powered incentive), and hence he could not gain from shirking by the other agent. Thus a discriminatory wage scheme plays a role by creating a “discrepancy” between the interests of agents and helping to deter their collusion.

Although there are several papers dealing with the problems of collusion in moral hazard environments, few studies characterize the properties of collusion-proof contracts when the principal uses relative performance evaluation. In different contexts Baliga and Sjöström [1] and Macho-Stadler and Pérez-Castrillo [10] show that delegation may be optimal when agents collude: The principal lets one agent contract separately with the other agent. Their paper shares a common feature with ours, which is the result that one agent is asymmetrically treated by the principal. However, the current paper has a different information structure and results. For example, Baliga and Sjöström [1] assume that the effort level one agent has chosen is commonly observable between agents and that side-contracts can be directly contingent on that effort, while we allow only monetary side-transfers. Moreover, they assume that the agents are asymmetric ex ante, while we derive asymmetric contracts with symmetric agents. Macho-Stadler and Pérez-Castrillo assume that all agents obtain their reservation payoffs in equilibrium, while we show that one agent can collect ex ante positive rent but the other agent cannot even when they are symmetric.

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3 See [3,5,8] for the issues of collusion in moral hazard models.
4 See also Faure-Grimaud et al. [4] for a related issue, although they have the model of adverse selection and asymmetric agents (a supervisor and an agent).
There is also a large body of economic analysis about discrimination. For example, Bowles [2] discusses how a “divide and conquer” strategy is introduced in firms to discriminate against workers who are otherwise identical with respect to their abilities. Our model also sheds light on this argument from the viewpoint of a principal concerned about collusion among agents. In this context, discrimination, or at least asymmetric treatment of ex ante identical agents, can arise endogenously.

The rest of the paper is organized as follows: in Section 2 we set up the basic model of a principal–multi-agent relationship. In Section 3, as a benchmark analysis, we characterize the optimal contract when agents cannot collude at all. In Section 4 we introduce the possibility of collusion between agents into the model. First, we show that a collusion-proof contract must be accompanied by low-powered incentives, as compared to a collusion-free situation. Second, we also show that a discriminatory wage scheme is derived as an optimal way of deterring collusion. Section 5 provides concluding remarks and discusses some extensions of the model.

2. The model

We consider an organization that consists of three risk neutral players, called the principal and the agents. We will adopt the convention that the principal is female and each agent is male. A principal hires two agents (agent $A$ and $B$) whose efforts stochastically yield some outputs to her. The output that agent $i$ generates is given by $y_i = e_i + e_i$ where $e_i$ is the effort agent $i$ has chosen and $e_i$ represents a random shock. Each agent chooses either high or low effort: $e_i \in \{h, l\}$, $i = A, B$, and choosing high effort $e_i = h$ costs him $c > 0$.\(^5\) The utility function of each agent is defined by $w/C_0(C(e))$ where $w$ is his income and $C(e)$ denotes the effort cost, i.e., $C(h) = c$ and $C(l) = 0$. We assume that the random variables $e_i$, $i = A, B$, are identically and independently distributed with a symmetric distribution function $F(\cdot)$ on $\mathbb{R}$, where $F$ is continuously differentiable and has the symmetric properties: $F(0) = 1/2$ and $F(x) = 1 - F(-x)$ for any $x \in \mathbb{R}$. The corresponding density function is denoted by $f(\cdot)$ where the symmetric assumption implies $f(x) = f(-x)$ for any $x \in \mathbb{R}$.

The effort level each agent has chosen is observable only to himself. Moreover, we assume that absolute measures of agents’ performances $y_A$ and $y_B$ are not available (they may be non-observable) but only their relative ranking is verifiable and hence contractible. This assumption may be justified, for example, when workers’ (agents’) performances are based on subjective evaluation by their supervisors and hence these absolute values may be more difficult to describe and verify in an unambiguous way than their relative ranking. Since our main concern is with the problem of how optimal contracts should be modified when collusion between the agents arises under relative performance evaluation, it is sufficient to restrict our attention to the

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\(^5\)In this paper we restrict our attention to a two-effort case and pure strategy equilibria. See [6] for some extensions of the model to allow more than two efforts and mixed strategies.
situations where the principal has no alternatives but to utilize the limited information about relative performances for giving the agents work incentives. Specifically we assume that it is publicly observable whether $y_i > y_j$, for $i, j = A, B, i \neq j$. Thus the contract the principal offers to agent $i$ should specify the payments to him contingent only on whether $y_i > y_j$. Let $W_i$ and $L_i$ denote the wages for agent $i$ to be paid when $y_i > y_j$ and when $y_i < y_j$ respectively. Then the principal offers a contract $\omega \equiv \{(W_i, L_i)\}_{i=1,2}$. We also assume the existence of limited liability so that the payments to any agent must be non-negative, i.e., $W_i \geq 0$ and $L_i \geq 0$ for $i = A, B$. Finally the reservation payoffs of all parties are assumed to be zero.

The probability distribution function of $y_i$ is given by $\Pr[y_i \leq y] = F(y - e_i)$. Furthermore, the distribution function of $e_i - e_j$ is also computed as

$$G(\delta) = \Pr[e_i - e_j \leq \delta] = \int_{-\infty}^{\infty} F(\delta + e)f(e)\,de.$$  

Note that $G(\cdot)$ has a symmetric property as $G(-\delta) = 1 - G(\delta)$, which follows from

$$1 - G(\delta) = \Pr[e_i - e_j > \delta]$$
$$= \Pr[\delta < e_i - e_j]$$
$$= G(-\delta).$$

We will consider the following timing of the game. Note that Stage 3 should be neglected if we are in the collusion-free situation.

1. The principal offers an initial contract $\omega$.
2. Each agent decides whether to accept it or not.
3. The agents may write a side-contract. We assume that the realizations of $y_A$ and $y_B$ are not observable to the agents at all. They can thus collude on side-transfers contingent only on observable (and verifiable) information, i.e., whether $y_A > y_B$ or not. We denote by $t_i$ the side-transfer paid by agent $i$ when $y_i > y_j$ (i.e. the wage $W_i$ is paid to agent $i$ by the principal).
4. The agents choose their efforts simultaneously.
5. The outputs $y_A$ and $y_B$ are realized and final payments are made according to initial and side-contracts.

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As long as the agents’ performances are ex post observable to the principal, the principal can still elicit work efforts from the agents even when their performances ($y_i$) are not verifiable, by attaching a higher wage to a particular job and promoting the better performing agent to that job through the principal’s ex post decisions for task assignment. This alternative model is analyzed in [6]. Of course, it is also important to consider how our results are changed when we allow contracts to be contingent on the absolute performances. Under the timing of side-contracts assumed in this paper, the principal may not stick to the use of relative performance evaluation when absolute performance measures are available and she faces collusion between the agents. We will discuss this issue in our concluding remarks.

If the outputs ($y_i$) are observable to the agents, their information may be exploited via a message game, although the outputs are not verifiable. To avoid such complexity, we assume that the outputs are not observable to the agents.
3. The collusion-free solution

First, as a benchmark, we examine the collusion-free situation where the agents cannot collude at all. Thus they will choose their efforts, following only the contract offered by the principal in the first stage. Recall that agent $i$ obtains the wages $W_i$ and $L_i$ when $y_i > y_j$ and when $y_i < y_j$ respectively.

The probability that $y_i > y_j$ occurs is given by

$$
\Pr[y_j < y_i] = \Pr[e_j - e_i < e_i - e_j] = G(e_i - e_j).
$$

Recall that the symmetric property of $F(\cdot)$ implies $G(0) = 1/2$ and $G(\delta) = 1 - G(-\delta)$ for any $\delta \in \mathbb{R}$.

The expected payoff of the principal is thus given by

$$
V(\omega; e_A, e_B) \equiv \sum_i E[y_i] - \sum_{i \neq j} G(e_i - e_j)(W_i + L_j).
$$

In the following we will maintain the assumption that it is optimal for the principal to elicit the high effort pair $e = (h, h)$ from the agents in the collusion-free situation. This assumption will hold if the effort cost $c > 0$ is not large.

Then the incentive compatibility (IC) must ensure that the high effort pair $e = (h, h)$ becomes a Nash equilibrium under a given initial contract $\omega = \{(W_i, L_i)\}$. Thus choosing $e = h$ maximizes the following expected payoff for each agent $i = A, B$:

$$
EU_i(e, h) \equiv G(e - h)W_i + (1 - G(e - h))L_i - C(e),
$$

where $C(h) = c > 0$ and $C(l) = 0$.

Defining $\Delta P \equiv G(0) - G(l - h)$,\footnote{By symmetry we also have $\Delta P = G(h - l) - G(0)$.} we obtain the (IC) constraints for implementing the high effort pair $e = (h, h)$ as follows:

$$
\Delta P \Delta w_i \geq c, \quad i = A, B
$$

(1)

where $\Delta w_i \equiv W_i - L_i$ denotes the wage premium for agent $i$. The left-hand side of (IC) represents the extra gain from choosing high effort as compared to choosing low effort when the other agent chooses high effort, while the right-hand side is the cost of doing so.

On the other hand, since $G(0) = 1/2$ by symmetry, the individual rationality constraints, which require that each agent must obtain at least the reservation payoff, are given by:

$$
\frac{1}{2} W_i + \frac{1}{2} L_i - c \geq 0, \quad i = A, B,
$$

(2)

provided that agents are induced to choose the high effort pair. Furthermore, the wages must satisfy the limited liability constraint:

$$
W_i \geq 0, \quad L_i \geq 0, \quad i = A, B.
$$

(3)
The principal chooses a contract $\omega$ to maximize his or her expected payoff $V(\omega; h, h)$ subject to (IC), (IR) and (LL). By (IC) and (LL), (IR) is slack and (IC) is binding. Thus the following result is immediate.

**Proposition 1.** The optimal collusion-free contract is uniquely determined by the following anonymous wage scheme:

$$W_i = \frac{c}{\Delta P}, \quad L_i = 0, \quad i = A, B.$$ 

The above result implies that the principal has no reasons to discriminate against the agents on their wage schemes, provided collusion is impossible. This result can be quite easily understood because both agents are identical with respect to their productive abilities.

In the next section we will undertake the analysis in the presence of collusion between the agents.

### 4. Collusion-proof contracts

#### 4.1. The impossibility of implementing the high effort pair

In this paper we employ a stringent notion of collusion proof as follows: to prevent the agents from colluding, the principal must ensure that the agents have no incentives to make side-transfers to each other on the equilibrium path for a given initial contract.\(^9\)

To understand this notion more precisely, consider an initial contract $\omega = \{W_i, L_i\}$ which is designed to induce the agents to choose the effort pair $(e^o_A, e^o_B)$ when they do not collude. That is, in the absence of collusion this contract gives agent $i$ the following expected payoff:

$$EU_i^o(e^i, e^j) = G(e^i - e^j)W_i + (1 - G(e^i - e^j)L_i - C(e^i)).$$

A side-contract between the agents specifies a pair of monetary transfers $(t_A, t_B)$ where $t_i$ is the transfer paid by agent $i$ when $y_i > y_j$. When this side-contract induces the agents to choose the effort pair $(\hat{e}_A, \hat{e}_B)$, which differs from $(e^o_A, e^o_B)$, agent $i$ obtains the following expected payoff:

$$EU_i^e(\hat{e}_i, \hat{e}_j) = G(\hat{e}_i - \hat{e}_j)(W_i - t_i) + (1 - G(\hat{e}_i - \hat{e}_j))(L_i + t_j) - C(\hat{e}_i).$$

The agents will agree on the side-contract if and only if the following constraints are satisfied. First, it must induce the prescribed effort pair $(\hat{e}_A, \hat{e}_B)$ as a Nash equilibrium outcome in the effort choice game because the agents cannot directly

\(^9\)In this definition there is no specification of the agents’ bargaining powers in the collusion subgame, and thus the principal is assumed to ensure that the initial contracts are robust to any such specification. The following results do not depend on this notion of collusion proof: they will hold even if we explicitly introduce the bargaining powers of the agents into the side-contracting game. See the remarks below after Propositions 2 and 3 for more detail.
collude on their effort choices. That is,
\[ \hat{e}_i \in \arg \max_e \ EU^e_i(e, \hat{e}_j), \quad i, j = A, B, \quad i \neq j. \] (4)

Second, it must improve both agents’ payoffs from the initial contract in the Pareto dominance sense: both agents are not worse off and at least one agent is strictly better off by agreeing on the side-contract. That is,
\[ EU^e_i(\hat{e}_i, \hat{e}_j) \geq EU^e_i(\hat{e}_i^0, \hat{e}_j^0), \quad i = A, B, \] (5)

where at least one inequality must be strict. Third, it must satisfy the limited liability constraints, where every agent cannot pay more than he obtains:
\[ W_i \geq t_i \geq -L_j, \quad i, j = A, B, \quad i \neq j. \] (6)

The initial contract \( \omega \) is said to be collusion proof if there exists no such side-contract on which the agents would both agree. That is, \( \omega \) becomes collusion-proof if there exist no side-transfers \((t_A, t_B)\) to satisfy all the above conditions (4)–(6) under \( \omega \).

Although it seems that we must deal with a quite large set of possible initial contracts, we can resort to the Collusion–Proof Principle in the setting of our model: The principal can restrict her attention only to collusion-proof initial contracts defined above without loss of generality. That is, for any initial contract which is not collusion proof, the principal can replicate the same outcome by some collusion-proof contract. Thus this principle permits us to look for the optimal initial contract only within the class of collusion-proof contracts instead of all possible contracts.

The intuition behind the principle is similar to the proof of the Revelation Principle. Suppose that some initial contract \( \omega^* = \{(W^*_i, L^*_i)\} \) implements the effort pair \((\hat{e}_i^*)\) through a side-contract \((t_i^*)\). Then the principal can design a new wage scheme \( \tilde{\omega} = \{(W^*_i - t_i^*, L^*_i + t_j^*)\} \), which implements the same effort pair and attain the same expected wage payment without collusion (see [6] for more formal proof).

We will now show that it is impossible to implement the high effort pair \((h, h)\) through a collusion-proof contract. From the analysis in Section 3, we know that any initial contract that implements \((h, h)\) must satisfy the (IC) constraints, \( \Delta P \Delta w_i \geq c \) for \( i = A, B \), where recall that \( \Delta P = G(h - l) - G(0) \) and \( \Delta w_i = W_i - L_i \). Suppose that \( \Delta w_A \geq \Delta w_B \) without loss of generality. We then show that the agents would agree on a side-contract \((t_A, t_B)\) that induces them to choose the effort pair \((e_A, e_B) = (h, l)\) and improves their payoffs from the initial contract.

To see this, consider the following side-transfer:
\[ \hat{t} \equiv t_A = t_B = \frac{\Delta P \Delta w_B - c}{2G(h - l) - 1}. \] (7)

This side-transfer is designed to make agent \( B \) (who is induced to choose low effort) just indifferent between accepting and rejecting it. That is, \( \hat{t} \) is the minimum bribe paid to agent \( B \) that will induce him to shirk instead of working hard, given agent \( A \) still works hard. Although each agent benefits from shirking by his peer (because he then collects the wage premium with higher probability), the fact that \( \Delta w_A \geq \Delta w_B \) implies that such a gain for agent \( A \) becomes larger than or equal to that for agent \( B \).
Thus agent $A$ would have the strict incentive to make the side transfer $\hat{t}$ to agent $B$ inducing him to shirk. This side-contract will then improve the agents’ payoffs.

To see this more precisely, we first show that the above side-contract $\hat{t}$ induces the agents to choose $(e_A, e_B) = (h, l)$. Under the side-contract $\hat{t}$ agent $B$ obtains the following expected payoff by choosing $e_B = l$, given $e_A = h$:

$$G(l - h)(W_B - \hat{t}) + (1 - G(l - h))(L_B + \hat{t})$$
$$= G(l - h)\Delta w_B + L_B + (2G(h - l) - 1)\hat{t}$$
$$= \frac{1}{2} \Delta w_B + L_B - c,$$

where the first equality follows from the symmetry of $G$ (i.e. $G(h - l) = 1 - G(l - h)$) and the final equality from the definitions of $\hat{t}$ and $\Delta P$.

Since agent $B$ would obtain the expected payoff $\left(\frac{1}{2}\right)\Delta w_B + L_B - c$ when both agents choose high efforts under the side-contract $\hat{t} = t_A = t_B$, the above condition shows that agent $B$ is indifferent for choosing between two efforts, given $e_A = h$.

Next consider the incentive of agent $A$. Under the side-contract $\hat{t}$ agent $A$ obtains the following expected payoff by choosing $e_A = h$, given $e_B = l$:

$$G(h - l)(W_A - \hat{t}) + (1 - G(h - l))(L_A + \hat{t}) - c$$
$$= G(h - l)\Delta w_A + L_A - (2G(h - l) - 1)\hat{t} - c$$
$$= G(h - l)\Delta w_A + L_A - (\Delta P \Delta w_B - c) - c$$
$$\geq \frac{1}{2} \Delta w_A + L_A,$$

where the second equality follows from the definition of $\hat{t}$ and the final inequality from $\Delta w_A \geq \Delta w_B$. The final expression of (9) also corresponds to the expected payoff of agent $A$ when he chooses $e_A = l$, given $e_B = l$. Thus, from (8) and (9) we conclude that the effort pair $(e_A, e_B) = (h, l)$ can be a Nash equilibrium in the collusion subgame under the side-contract $\hat{t}$.

Second, we show that the side-contract $\hat{t}$ Pareto dominates the initial contract. Note that agent $i$ would obtain the expected payoff $\left(\frac{1}{2}\right)\Delta w_i + L_i - c$ under no collusion because the (IC) constraints are satisfied. Thus agent $B$ obtains the same expected payoff under collusion as that under no collusion. This directly follows from the above equalities (8) describing the incentive of agent $B$. Furthermore, the above inequalities (9) describing the incentive of agent $A$ imply

$$G(h - l)(W_A - \hat{t}) + (1 - G(h - l))(L_A + \hat{t}) - c$$
$$> \frac{1}{2} \Delta w_A + L_A - c$$

which shows that agent $A$ is also strictly better off by the side-contract $\hat{t}$ as compared to no collusion.
Finally the limited liability constraints are satisfied:

$$-L_i \leq 0 \leq \hat{i} = \frac{\Delta P \Delta w_B - c}{2G(h-l) - 1} = \frac{\Delta P \Delta w_B - c}{2\Delta P} < \Delta w_B \leq W_i$$

(11)

for \( i = A, B \), where the second inequality follows from \( \Delta P \Delta w_B \geq c \) and \( G(h-l) > 1/2 \), the second equality from \( 2G(h-1) - 1 = 2\Delta P \) and \( G(0) = 1/2 \), and the final inequality from \( \Delta w_A \geq \Delta w_B \) and \( L_i \geq 0 \) for \( i = A, B \) respectively.

The above argument shows the following result.

**Proposition 2.** There exist no collusion-proof contracts that implement the high effort pair \( e = (h, h) \).

Proposition 2 still remains true even if we specify the bargaining powers of the agents in the side-contracting subgame as long as we focus on the bargaining game which always results in an agreement when gains from collusion exist. This is simply because in Proposition 2 we show that there always exists a side-contract Pareto dominating any initial contract which satisfies (IC), (IR) and (LL).

The implication of Proposition 2 is that the optimal contract should be low-powered when agents collude. This is intuitive because attempting to obtain high efforts from both agents costs them too much and provides the opportunity for collusion between them.

The following corollary follows from Proposition 2.10

**Corollary 1.** The optimal collusion-free contract is not collusion proof if the effort cost \( c \) is sufficiently small.

4.2. Discriminatory wage schemes

We will now examine which initial contracts are collusion proof and what is optimal in those collusion-proof contracts. From Proposition 2 we know that the high effort pair \( e = (h, h) \) cannot be implemented. Since it is obvious that the low effort pair \( e = (l, l) \) is implemented by a trivial collusion-proof contract \( W_i = L_i = 0 \) for all \( i \), we will look for the contracts to implement the remaining effort pair \( (e_A, e_B) = (h, l) \) or \( (e_A, e_B) = (l, h) \). By symmetry we will consider whether or not \( (e_A, e_B) = (h, l) \) is implemented.

First, we define the set of constraints that implement the effort pair \( (e_A, e_B) = (h, l) \), provided the agents cannot collude at all. These constraints are given as follows:

\[
\Delta P \Delta w_A \geq c, \quad \Delta P \Delta w_B \leq c, \\
G(h-l)W_A + (1 - G(h-l))L_A - c \geq 0, \\
G(l-h)W_B + (1 - G(l-h))L_B \geq 0,
\]

(12)\hspace{1cm}(13)\hspace{1cm}(14)

in addition to (LL).

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10 Recall that implementing the high effort pair \( e = (h, h) \) becomes optimal in the collusion free situation when the effort cost \( c \) is small enough.
The first constraint in (12) means that high effort is elicited from agent A, while the second one induces agent B to choose low effort. Conditions (13) and (14) represent the individual rationality constraints for agent A and B respectively. Then it is obvious that the optimal solution becomes $W_A = c/\Delta P$ and $W_B = L_A = L_B = 0$ when we consider the problem of maximizing $V(\omega; h, l)$ subject to (12)–(14) and (LL). Let $\omega^*$ denote this optimal contract. Let also $\hat{V}_{hl}$ be the maximum payoff of the principal at this optimum.

On the other hand, the principal can obtain the payoff $\hat{V}_{hl} = 2l + \sum_i E[e_i]$ when she implements the low effort pair $e = (l, l)$ by using the trivial contract $W_i = L_i = 0$ for $i = A, B$. The following assumption avoids the trivial case that implementing $e = (l, l)$ becomes the overall optimum.

**Assumption P.** $\hat{V}_{hl} > \hat{V}_{ll}$

Given Assumption P, the principal will implement the asymmetric effort pair $(h, l)$ by using the discriminatory wage $\omega^*$, provided this is collusion-proof. The intuition why $\omega^*$ is collusion-proof is that the agent, say A, who is offered the high powered incentive scheme (i.e., $W_A = c/\Delta P$ and $L_A = 0$) would not agree on side-contracts inducing him to choose low effort unless the expected side-transfer he collects is large enough to compensate him for the lower probability of obtaining the high wage premium. However, since the other agent, B, faces zero wage regardless of the outcome, he would be strictly worse off by making such payment to agent A.

More precisely, if the initial contract $\omega^*$ is offered to induce the effort pair $(h, l)$, there are two possibilities for collusion: One is that the agents agree on a side-contract to induce them to choose low efforts $(e_A, e_B) = (l, l)$. The other is that they collude on the effort pair $(e_A, e_B) = (l, h)$. By virtue of Proposition 2 and the Collusion-Proof Principle, we do not have to consider the remaining set of side-transfers that induce the high effort pair $e = (h, h)$. This follows from the following fact: if an equilibrium exists in which the agents choose $e = (h, h)$ via some side-contract, under the Collusion-Proof Principle a collusion-proof contract must also exist that implements the same effort pair $e = (h, h)$ without collusion. However this contradicts Proposition 2.11

Fix $\omega^*$ as an initial contract and first consider the case where the agents are colluding on $(l, l)$ through some side-contract $(t_A, t_B)$. Agents A and B will agree on this side-contract only if

$$\frac{1}{2}(W_A - t_A) + \frac{1}{2}t_B \geq G(h - l)W_A - c,$$

$$-\frac{1}{2}t_B + \frac{1}{2}t_A \geq 0,$$

11 Note that such a collusion-proof contract satisfies all (IC), (IR) and (LL) because by definition it induces the effort pair $e = (h, h)$ as an equilibrium.
where at least one of these inequalities must be strict. Summing them, we obtain
\[
\frac{1}{2} W_A > G(h - l) W_A - c
\]
which however cannot hold due to the definition of \( W_A \), i.e., \( W_A = c/\Delta P \), and
\( \Delta P = G(h - l) - 1/2. \)

Next consider the case where the agents are colluding on \((l, h)\) through some side-contract \((t_A, t_B)\). Agents \( A \) and \( B \) will agree on this side-contract only if
\[
G(l - h)(W_A - t_A) + (1 - G(l - h))t_B \geq G(h - l) W_A - c,
\]
\[
G(l - h)t_A - (1 - G(l - h))t_B - c \geq 0,
\]
where at least one of these inequalities must be strict. Summing them, we obtain
\[
G(l - h) W_A > G(h - l) W_A
\]
which however cannot hold because \( W_A > 0 \) and \( G \) is increasing.

The above argument yields the following our main result.

**Proposition 3.** Suppose that Assumption P holds. Then the optimal collusion-proof contract is given by the non-anonymous wage scheme \( W_i = c/\Delta P \) and \( L_i = W_j = L_j = 0 \), \( i, j = A, B, i \neq j \). In other words, one agent is offered a higher-powered incentive scheme than the other even when they are symmetric.

Proposition 3 still also holds even if we take into account the bargaining powers of the agents in the side-contracting subgame. Since we show that there exist no side-contracts Pareto dominating the initial contract \( \omega^* \), no gains from collusion are left for the agents.

Proposition 3 also implies that one agent can collect some positive rent under the optimal collusion-proof contract \( \omega^* \) but the other cannot.

**Corollary 2.** Suppose that Assumption P holds. Then one agent is favored by the principal in that he obtains the positive rent \( U \equiv cG(h - l)/\Delta P - c > 0 \) but the other does not under the optimal collusion-proof contract \( \omega^* \).

Some implications of discrimination in organizations are derived from the above results. For example, suppose that \( W_i \) includes not only the wage attached to the current job on which agent \( i \) is working but also future incomes associated with his career path. Thus the higher wage \( W_i \) may reflect a better future promotion opportunity: for example, if \( W_A > W_B \), agent \( A \) has a better opportunity for future promotion than agent \( B \). Which agent is treated better is determined only by their identities (for example, their sex). This may explain the cases often observed in the real world where workers are discriminated against with respect to their career opportunities even when they have the same productive abilities.
5. Concluding remarks

In this paper we have developed a principal–multi-agent model to understand how a discriminatory wage scheme is introduced as an optimal way of deterring collusion between the agents. The principal has an incentive to create a “discrepancy” between the interests of the agents by offering a discriminatory wage scheme. Such discrimination makes collusion between them difficult, and hence the optimal collusion-proof contract becomes non-anonymous.

We conclude the paper by briefly discussing several extensions of the basic model. First, as noted in footnote 5, we can extend the model to allow the agents to use mixed strategies and choose more than two efforts (see [6] for more detail). Second, at the outset we ruled out the use of contracts contingent on the absolute measures of agents’ performances, by assuming that they are not verifiable. As we argued earlier, this assumption may be justified, for example, when workers’ performances are based on subjective evaluation by their supervisors and hence these absolute values are more difficult to verify than their relative ranking. It is interesting to consider how the analysis presented in this paper would be changed when we allow contracts to be contingent on absolute performance measures. If this is possible in the setting of our model, the principal may not stick to the use of relative performance evaluation because doing so raises the possibility of collusion between the agents and hence hurts the principal. However, we cannot generally conclude from this observation that the principal should always abandon the use of relative performance evaluation when absolute performance measures are available and agents collude. Indeed Ishiguro and Itoh [7] show that the principal can implement the first best solution by using relative performance evaluation when the agents can write a side-contract after they have chosen their efforts and mutually observe these efforts, although this case is not covered by this paper. Thus whether relative performance evaluation is effective or not depends on the timing of side-contracts and the information structure. All we can say at this time is that a discriminatory wage scheme is more likely to be introduced when it is too difficult and costly to use absolute performance evaluation, and collusion among agents arises.

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