Optimal Timing of Firm's R&D Investment under Incomplete Information: A 
Real Options and Game-theoretic Approach

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ABSTRACT
In a real options and game-theoretic framework, this paper examines the optimal R&D investment timing of an incumbent (firm 1) under uncertainty, which faces the threat of preemption by a potential entrant (firm 2). We incorporate incomplete information into the model by assuming that the incumbent does not know the entrant's investment threshold but know its distribution. We find that incomplete information restrains the competition from undermining the waiting option value and delays the incumbent's R&D investment. The incumbent's optimal R&D investment timing is determined by its conjecture about entrant's hazard rate, which measures the degree of entrant's threat of preemption. The more the hazard rate is, the earlier the firm invests.

Keywords: Optimal timing, R&D investment, Real options, Game theory, Incomplete information

1. INTRODUCTION
In the uncertain and competitive high-tech industry, one of the most important decisions of firms is when to invest in R&D, i.e. the R&D investment timing decision. A standard framework for the investment timing decision is the real options approach, which assumes that the opportunity to invest in R&D is analogous to an American call option on the investment project, and the timing of investment is economically equivalent to the optimal exercise decision for an option. In traditional real option modeling, the optimal exercise problem is always modeled as isolated optimal stopping problem without strategic interactions. Therefore, in real options' view, it is optimal to delay exercising the option to invest, even when it would be profitable to do so at once, in the hope of gaining a higher payoff in the future⁴. However, real options, unlike their financial counterparts, are rarely backed by legal contracts guaranteeing the holder’s rights in precise terms. There are always other firms that also have access to the non-proprietary investment opportunity. When real options are held by a small number of competitive firms with an advantage to the first mover, each firm has incentive to invest early and its ability to delay is undermined by the fear of preemption. In order to deal with the tension between real options and strategic competition, this paper adapts the option game-theoretic approach that merges real options with game theory.

The study of option game theory started from the seminal paper of Smets³, in which he studies the foreign direct investment decision in the duopoly market, considering both the exchange rate uncertainty and strategic interaction between two competitive firms. Dixit and Pindyck summarize this model in their outstanding real option textbook³. They assume that firms are not active in the market at original time and the firms as leader or follower is given exogenously. There is only preemptive equilibrium. Huisman and Kort extend this model through introducing mix strategy equilibrium and assuming that firms are active in the market at original time, i.e. the profit cash flows are positive even they don’t invest yet⁴. Grenadier analyses real estate development using continuous-time, leader–follower games in which firms strategically choose threshold points for their investments⁵. Weeds considers irreversible in competing R&D projects with uncertain returns under a winner-takes-all patent system⁶. Huisman and Kort study the optimal timing of technological innovation of a single firm in a duopoly framework. They examine the optimal technology investment decision of an individual firm, while taking into account the possible occurrence of better technologies in the future and competition of other firms⁷. Smit and Ankum⁸ and Trigeorgis⁹ study strategic investment decisions by different firms using binomial models. Huisman⁹ and Pawlina and Kort¹⁰ analyze the situation where two firms' investment costs are asymmetric and one has cost advantage over another.

In the all above literature, they assume that the information is complete. Although clearly the right starting point for analysis, this approach has two limitations. First, the assumption of complete information is often unrealistic. A significant risk for many firms is that their conjectures about the investment cost and timing of competitors will prove incorrect. Second, the investment cost, timing and size of firms are often private information when firms make investment decision. The literature considering incomplete information is limited, while some recent papers have started analyzing the exercise of real options under alternative information structures. Lambrecht and Perraquin¹¹ introduce incomplete information and preemption into an equilibrium model of firms that have the opportunity to enter into a new market. Firms know their own cost of entry but only know the distribution of their competitor's investment costs. They show that the optimal investment threshold may lie anywhere between the zero-NPV threshold and
the firm’s optimal monopolistic and non-competitive threshold, depending on the fear of preemption implied by the distribution of the competitors costs. Furthermore, higher product market uncertainty leads to more delay, conform to what is predicted by the real options paradigm. Grenadier[13] describes a model in which firms learn about the investment payoff from the actions of other agents, i.e. each firm has a private signal about the payoff of the investment that is revealed when it acts. Information revelation allows firms that have not yet acted to update their information about the value of the underlying investment.

The model closest to ours is by Lambrecht and Perraudin, but in their model, they assume the two firms have the same market position and have the same possibility of being leader or follower, while our paper focuses on the optimal R&D investment timing of the incumbent, which has monopolized position in the market, under the threat of the potential entrant.

The remainder of the paper is organized as follows. In Section 2, we present the basic model and examine the optimal R&D investment timing of the incumbent under incomplete information. Section 3 investigates the impact of incomplete information and hazard rate on the R&D investment timing. Concluding remarks are given in Section 4.

2. THE MODEL

Two risk-neutral firms, an incumbent (firm 1) and a potential entrant (firm 2), have the opportunity to invest in competing R&D projects. Project is directly competitive: the firms strive for the same patent and competitive advantage. Specifically, the incumbent is serving a monopolized market under the threat of a competitive: the firms strive for the same patent and competing R&D projects. Project is directly

2.1 The Optimal Timing of Monopoly Firm

As an analysis benchmark, we first give the Marshallian threshold \( Y_{1\text{Mar}} \) and the monopoly threshold \( Y_{1M} \). We follow Dixit[3] in defining the Marshallian threshold, \( Y_{1\text{Mar}} \), as the point at which the total expected discounted income flow equals the cost of investment:

\[
Y_{1\text{Mar}} = (r-\alpha)I \tag{2}
\]

In terms of real option approach, the incumbent’s monopoly threshold \( Y_{1M} \), where there is not any threat of the entrant, is:

\[
Y_{1M} = \frac{\beta}{\beta_1-1} (r-\alpha)I \tag{3}
\]

where \( \beta_1 \) is the positive root of the quadratic equation \( \frac{1}{2}\sigma^2\beta^2 + (\alpha - \frac{1}{2}\sigma^2)\beta - r = 0 \), and \( \beta_1 > 1 \). For \( r > \alpha \) and \( \beta_1 > 1 \), \( Y_{1M} > Y_{1\text{Mar}} \), this means that the incumbent has incentive to delay invest in R&D under uncertainty for the waiting option value. This is the basic conclusion of real option approach.

2.2 The Optimal Timing of Firm with Threats of Preemption

Assuming \( Y_{2p} \) is the entrant’s optimal investment threshold, and the corresponding optimal investment timing is \( T_{2p} = \inf\left(t\mid Y \geq Y_{2p}\right) \), which means the time that \( Y \) first hits \( Y_{2p} \). To introduce incomplete information, we assume that the incumbent conjectures that the entrant invests when \( Y \) first crosses some level \( Y_{2p} \), and that \( Y_{2p} \) is an independent draw from a distribution \( F(x) \), which is a cumulative possibility function, \( 0 \leq F(x) \leq 1 \), and has a continuously differentiable density \( f(x) = F'(x) \) with positive support on an interval, \([Y, Y_c]\).

The structure of learning implied by our assumptions is quite simple. Since the entrant invests only when \( Y \) first crosses a threshold, the incumbent learns about the entrant when \( Y \) hits a new higher value, \( \bar{Y} \), within the interval \([Y_L, Y_U]\). When this happens, if the entrant invests, the incumbent learns that the entrant’s threshold level is the current value, \( \bar{Y} \). Conversely, if the entrant does not invest, the incumbent learns that the entrant’s threshold lies in a higher range of \( \bar{Y} \) values than it had previously believed, i.e. \( Y_{2p} \in [\bar{Y}, Y_U] \). Thus, the incumbent’s conditional conjecture about the distribution of the entrant’s threshold, is:

\[
F\left(Y_{2p} \mid \bar{Y}\right) = P\{Y \leq Y_{2p} \mid Y \geq \bar{Y}\}
\]

\[
= \frac{F(Y_{2p}) - F(\bar{Y})}{1 - F(\bar{Y})} \tag{4}
\]

\[
\bar{Y} = \sup_{0 \leq x \leq 1} \{Y(x)\} \tag{5}
\]

\[
F\left(Y_{2p} \mid \bar{Y}\right)
\]

is the possibility of the entrant with investment threshold \( Y_{2p} \) holding option (i.e. no investing) when \( Y \geq \bar{Y} \). So, \( 1 - F\left(Y_{2p} \mid \bar{Y}\right) \) is the possibility of the incumbent preempted by the entrant that has not invested yet. We defined the hazard rate:

\[
h\left(Y_{2p} \mid \bar{Y}\right) = \lim_{\varepsilon \to 0} \frac{P\{Y_{2p} \leq Y \leq Y_{2p} + \varepsilon \mid Y \geq \bar{Y}\}}{\varepsilon} \tag{6}
\]

\[
= \frac{f\left(Y_{2p}\right)}{1 - F\left(Y_{2p}\right)}
\]
Given the distribution of the entrant's investment threshold, \( F(x) \), \( h(Y_{1,i} | F) \) is the possibility of preemptive investment by the entrant, i.e. \( h(Y_{1,i} | F) \) is the incumbent's conjecture about the degree of entrant's threat of preemption. As we know, the hazard rate is no relation with current value \( F \) because \( Y_{2,i} \geq F \).

The incumbent's value under the threat of preemption, \( V_{1,i} \), depends not just on the publicly observable profit variable \( Y \) but also on \( F \), i.e. \( V_{1,i} = V_{1,i}(Y, F) \). The \( F \) is a sufficient statistic for all that the incumbent has learnt about the entrant by time \( t \). The current value \( F \) only changes intermittently and is a finite variation, monotonously increasing process. When \( F \) does not change, according to the standard real option model\(^{(3)}\), the incumbent's value is:

\[
V_{1,i}(Y) = \left( \frac{Y_{1,i}}{r - \alpha} - I \right) \left( \frac{Y}{Y_{1,i}} \right)^{\alpha} \left[ 1 - F(Y_{1,i} | F) \right]^{\beta}
\]  

(7)

From (2), we know that, when \( \bar{Y} \leq Y \leq Y_{1,i} \), the possibility of the incumbent preempted by the entrant that has not invested yet is \( 1 - F(Y_{2,i} | F) \), and the incumbent's value must be adjusted by this possibility\(^{(3)}\):

\[
V_{1,i}(Y, \bar{F}) = \left( \frac{Y_{1,i}}{r - \alpha} - I \right) \left( \frac{Y}{Y_{1,i}} \right)^{\alpha} \left[ 1 - F(Y_{1,i} | \bar{F}) \right]^{\beta} \left[ 1 - F(Y_{2,i} | F) \right]
\]

(8)

Taking the derivative of \( V_{1,i} \) with respect to \( Y_{1,i} \) and setting it equal to zero, we obtain the first-order condition of the optimal threshold:

\[
\frac{1}{r - \alpha} - \frac{\beta}{Y_{1,i}} \left( \frac{Y_{1,i}}{r - \alpha} - I \right) - \frac{f(Y_{1,i})}{1 - F(Y_{1,i} | \bar{F})} \left( \frac{Y_{1,i}}{r - \alpha} - I \right) = 0
\]

(9)

As we defied, the hazard rate is \( h(Y_{1,i}) = f(Y_{1,i})/(1 - F(Y_{1,i} | \bar{F})) \). Substituting for \( h(Y_{1,i}) \) into equation (6) and rearranging it:

\[
Y_{1,i}^2 + \frac{\beta}{h(Y_{1,i})} - (r - \alpha) I \left( \frac{Y_{1,i}}{r - \alpha} - I \right) = 0
\]

(10)

Solving the equation (10), we can obtain the optimal investment threshold \( Y_{1,i} \).

Now, we prove that the equation (10) has a single positive solution \( Y_{1,i} \). Let the right hand of the equation (10) equal function \( g(Y_{1,i}) \), and from (2) and (3), we know \( g(0) < 0 \), \( g(Y_{1,M}) > 0 \), \( g(Y_{1,Mar}) < 0 \). Therefore, the equation (10) has a single positive solution \( Y_{1,i} \), and:

\[
Y_{1,Mar} \leq Y_{1,i} \leq Y_{1,M}
\]

(11)

From (11), we know that the incumbent's optimal investment threshold lies between the Marshallian threshold \( Y_{1,Mar} \) and the monopoly threshold \( Y_{1,M} \). This shows that the incomplete information reduces the erosion of waiting option value by the competition, and therefore waiting is still valuable even in the presence of preemption and competition.

### 3. DISCUSSION

In this section, we give a numerical solution of (10), and analyses the impact of hazard rate \( h(Y_{1,i}) \) on the incumbent's investment threshold \( Y_{1,i} \). Because the hazard rate \( h(Y_{1,i}) \) is also function of the threshold \( Y_{1,i} \), we need know the specific distribution \( F(x) \). Assuming the \( F(x) \) follows exponential distribution:

\[
F(x) = 1 - e^{-\lambda x}
\]

(12)

So its hazard rate is:

\[
h(x) = \lambda
\]

(13)

That is, the hazard rate of exponential distribution is constant, and we can let \( h(Y_{1,i}) = h \). In other words, if the entrant's threshold follows the exponential distribution, the hazard rate of preemptive investment is irrelative with \( Y_{1,i} \). Therefore, the threat of the entrant that the incumbent may face at any possible threshold can measured with the single constant \( \lambda \).

Now, we give a numerical solution of (10). The base case parameters are: \( \alpha = 0.01 \), \( \sigma = 0.2 \), \( r = 0.06 \), \( I = 100 \), \( \lambda = 1.5 \). From (2) and (3), we have \( Y_{1,Mar} = 5 \), \( Y_{1,M} = 10 \). The Fig. 1 is the curve of function \( g(Y_{1,i}) \), and the right point of intersection of \( g(Y_{1,i}) \) and horizontal axis is the incumbent's investment threshold \( Y_{1,i} \), and in this case \( Y_{1,i} = 5.54 \).

![Fig. 1 The incumbent's optimal investment threshold](image)

Furthermore, we analyses the impact of hazard rate \( h(Y_{1,i}) \) on the incumbent's threshold \( Y_{1,i} \). Form (10), when \( Y_{1,Mar} \leq Y_{1,i} \leq Y_{1,M} \), we have:

\[
\frac{dY_{1,i}}{dh} < 0, \quad \frac{d^2Y_{1,i}}{dh^2} < 0
\]

(14)

Therefore, we conclude that if the entrant's investment threshold follows the exponential distribution, the incumbent's optimal investment threshold is determined by its conjecture about entrant's hazard rate: the more its
hazard rate is, the less the incumbent's investment threshold is (see Fig.2).

![Fig.2 The effect of hazard rate on the incumbent's investment threshold](image)

4. CONCLUSION

We examine the optimal R&D investment timing of an incumbent under uncertainty, which faces the preemptive threat of a potential entrant. We incorporate incomplete information into the model by assuming that the incumbent does not know the entrant’s investment threshold but know its distribution. We find that the incumbent's optimal investment threshold lies between the Marshallian threshold and the monopoly threshold. This shows that the incomplete information reduces the erosion of waiting option value by the competition, and therefore waiting is still valuable even in the presence of preemption and competition. Furthermore, we show that the hazard rate of the entrant has an impact on the incumbent's optimal timing: the more the hazard rate is, the earlier the firm invests.

For sake of space, we focus on only the incumbent's optimal R&D investment timing under the threat of the potential entrant, and not discuss the entrant's optimal R&D investment timing. The model could be extended in a number of ways. First, we can examine the equilibrium of the two competitive firm's strategic interaction under incomplete information. Second, we can extend the model to oligopoly market. Finally, we can analyze the situation where there are both cost advantages and information advantages, et al.

REFERENCES